Dynamic quality regulation of the electricity grid
An intertemporal optimization model for analyzing path-dependencies in the provision of infrastructure quality

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Abstract

Climate change imposes new challenges on electricity transportation infrastructure due to higher frequencies of extreme weather events. To maintain continuity of supply and avoid high macroeconomic costs associated with electricity blackouts investments in quality are needed. This paper aims at fostering the understanding of quality investments if path dependencies occur and associated consequences for optimal regulation.

In our intertemporal optimization model a monopolist supplies a single service with fixed capacity and a variable quality to a grid user, who demands capacity. The model is formulated in continuous time and captures the investment behavior in quality generalized for different regulatory regimes. We find that bistability may occur if depreciation is monotonically decreasing in quality.

By using this assumption, our model add to the existing static analysis in several ways: The results indicate that quality provision is highly dependent on the initial quality level if parameter values cause bistability. Due to path dependencies it may be optimal in the long-run to stick to low quality levels as investment efforts are to high compared to gains from quality increases. However, in the short-run high quality levels are optimal. Therefore, the regulator faces a trade-off between long-run and short run optimality. Furthermore, by evaluating the optimality of the unregulated case, the absolute value of welfare loss due to missing regulation depends on the initial quality level, if bistability occurs in one of the two cases.

The results are relevant for the practice of regulation. As quality regulation in natural monopolies can produce path dependencies, infrastructure quality can lock-in at short-run socially inferior levels. In face of non-standard depreciation rates common regulatory approaches might not be able to enforce the socially desired investments in infrastructure quality in the short-run. Thus, it might be feasible to overcome path dependencies by offering greater incentives for quality provision to achieve high quality levels.

JEL-Classification: C61, L15, L51

Keywords: dynamic optimization; electricity grid; natural monopoly; nonlinear dynamics; quality regulation; renewables;
1 Introduction

Climate change imposes new challenges on electricity transportation infrastructure. Increases in the frequency of extreme weather events Schaeffer et al. (2012); Mideksa and Kallbekken (2010); Ward (2013) will put more and more stress on electricity infrastructure in addition to growing requirements in regard to flexibility and capacity due to changes in production and consumption. To maintain continuity of supply and avoid high macroeconomic costs associated with electricity blackouts Ward (2013); Bliem (2005); Newman et al. (2011); Corwin and Miles (Corwin and Miles) investments in quality are needed. Economies have to provide incentives for grid providers to establish a resilient electricity infrastructure capable of withstanding future stresses. Therefore, the issue of quality regulation that incentives investments in infrastructure quality ranks high on the political agenda. Simultaneously, it received increasing attention in the scientific regulation literature to obtain a better understanding of relevant driving mechanisms in regard to quality investments.

Up to now, most of the literature concentrated on consequences of different regulatory policies on the optimality of quality provision (de Fraja 2008; Sheshinski, 1976; Spence, 1975), verifiability of quality (Lewis and Sappington, 1991; 1992) and empirical studies on quality regulation (Schmidthaler et al., 2015; Diekmann et al., 2007; Elliott, 2006; Sappington, 2005; Ajodhia and Hakvoort, 2005). It has been observed that cost-based approaches such as rate-of-return regulation are superior to incentive-based ones in regard to quality and usually quality is either under- or oversupplied (Spence 1975; Sheshinski 1976). Thus, it has been empirically observed that most European countries introduced an additional quality element into the regulatory formula (Schmidthaler et al., 2015) that finds its theoretical counterpart in the quality adjusted Vogelsang-Finshinger mechanism (de Fraja 2008). However, little attention has been put on the differences of quality and capacity provision in general and on the dynamics of quality provision. From environmental models it is known, that the quality of ecosystems such as lakes can be subject to unpredictable changes. Tipping points give rise to path dependencies causing look-ins into undesirable system states. To the best of my knowledge, it has not been elaborated systematically if these phenomena may also occur in regard to quality provision of infrastructure. However, these phenomena might be important on various levels. On the one hand, in regard to electricity infrastructure regulation might have very different effects.
in the face of path dependencies that cause multistability. On the other hand, external shocks associated with extreme weather events may cause look-ins into undesirable quality states. Therefore, it is the primary goal of this paper to analyze whether path dependencies may occur in the provision of infrastructure quality. Likewise, the associated differences in regard to the social optimality of regulation must be examined.

To this end, we present a dynamic model in continuous time that captures the investment behavior in quality generalized for different regulatory regimes. We study a situation where a monopolist supplies a single service with a verifiable quality subject to an exogenous set price-cap. Regulation is captured by a generalized formula that allows to compare the effects of different regulatory regimes on investments quality in a consistent setting. It takes into account cost-based and incentive-based regulatory approaches. Using this mathematical formulation the decisions of the regulated monopolist can be studied: She faces a dynamic optimization problem, in which costs of current investments need to be balanced against the regulated future returns on these investments. The investment behavior is derived for four different scenarios that allow for a consistent comparison of optimality in regard to the social planner, the unregulated monopolist and the regulated monopolist.

I find that bistability may occur if depreciation is monotonically decreasing in quality. This can be empirically justified in the light of increases in damages on infrastructure due to a higher frequency of blackouts expressed by quality. Even by using this assumption, two well known results from the literature can be derived in an dynamic setting: Firstly, the monopolist provides quality according to the marginal consumer, whereas the social planner takes the average consumer into account and thereby over- or undersupplies quality compared to the social optimum. Secondly, if quality is verifiable and the willingness to pay for quality can be observed by the regulator, she can always select parameters for regulation in order to obtain the long-run socially optimal level of quality. In addition, the results indicate that quality provision is highly dependent on the initial quality level if parameter values cause bistability. Due to path dependencies it may be optimal in the long-run to stick to low quality levels as investment efforts are to high compared to gains from quality. However, as long-run and short-run optimum differ, it may be feasible for the regulator to deviate from socially optimal long-run regulation to encourage investments in high quality levels, that are optimal in the short-run. Furthermore, by comparing the social planner and the
unregulated monopolist case, the welfare loss due to missing regulation may depend on the initial quality level, if bistability occurs in one of the two cases. The results are complemented by considerations on binding regulation. Dependent on the willingness to pay for quality, regulation may become non-binding and for certain parameters no steady state might exists.

These results are relevant for the practice of regulation. In face of high macro-economic cost of power outages on the one hand and the costly provision of quality on the other hand it is crucial for the regulator to enforce the socially desired quality. However, as quality regulation in natural monopolies can produce strong path dependencies, infrastructure quality can lock-in at short-run socially inferior levels. In face of non-standard deprecation rates common regulatory approaches might not be able to enforce the socially desired investments in infrastructure quality.

The remainder of this paper is structured as follows. The paper starts with a short literature overview on quality regulation and path dependencies. In Section 3.1 these insights are extended by theoretical considerations on how to model quality and on the conditions needed to obtain path dependencies. The results serve as ground for the derivation of a mathematical modeling framework in 3.2.2 and associated scenarios for analysis. In Section 4 the model is evaluated. It starts with a general deviation of steady state conditions and propositions for the equivalence of scenarios are obtained. On ground of these conditions a stability analysis of the model is conducted and the resulting optimal investment behavior is interpreted. Reflections on in regard to optimality in comparison of the different scenarios are conducted in section 4.3. Considerations on welfare are provided and both short-run consumer and producer surplus are calculated. The evaluation is finalized with an analysis of necessary conditions for binding regulation. The last two chapters provide a critical discussion of these results and give an outlook on how to proceed. All formal proofs are contained in the Appendix.

2 Quality regulation in the literature

Regulation of natural monopolies has a long standing history in economics. The following description draws from the introductory literature, which gives an extensive overview on regulation theory such as Armstrong and Sappington (2007) or Diekmann et al. (2007). In short, a natural monopoly occurs if a producer faces
sub-additive cost structures. In this case, it becomes socially optimal to provide
the good by only one producer. Usually this is the case for markets with high
shares of fixed costs, which are not divisible: If only one producer bears these
fixed costs she can provide any amount of goods at a cheaper price compared to
a variety of producers providing the same amount and each being exposed to the
full amount of fixed costs. However, in a monopoly situation without regulation
the profit-maximizing monopolist does provide a sub-optimal level of quantity at
prices higher than the optimal price due to missing competition. Therefore, regu-
lation aims at adjusting these price distortions towards the social optimum. Since
the average cost curve generally lies above the marginal cost curve, marginal cost
pricing, which is the social optimum does not provide incentives for monopolists
to enter the market. To solve this problem, regulation targets the second-best
solution, so called Ramsey prices, to account for cost recovery of fixed assets
investments undertaken by the monopolist.

Various kinds of regulatory approaches have been developed, each taking into
account further problems such as information asymmetries, transaction costs, in-
centives etc.. In general, tow main branches can be distinguished: Cost based ori-
entated and incentive orientated regulatory approaches, whereas cost-plus (CP)
or rate-of-return (RoR) regulation belongs to the former and price-caps (PC) or
revenue-caps (RC) or yardstick regulation (YR) to the latter (Schmidthaler et al.,
2015).

The original literature on regulation theory was mainly concerned with quan-
tity provision, leaving issues of service quality aside. This gap was addressed by
the works of Spence (1975) and Sheshinski (1976) who analyzed the provision
of quality under different regulatory regimes. Taking quality into consideration,
prices have to reflect not only the willingness to pay for quantity but also for
quality and related questions. Spence (1975) showed that the monopolist may
provide an efficient level of quantity in a cost based regulatory framework, but
another type of market failure occurs, if the marginal value of quality depends on
the quantity purchased: Quality is provided according to the preferences of the
marginal consumer, whereas for the socially optimal quality level, the average
consumer would have been taken into account. Thus, depending on the demand
structure, quality is either over- or undersupplied. A similar result was obtained
by Sheshinski (1976) for incentive regulation, where quality is generally under-
supplied, since it is not reflected in the price cap: An increase in quality leads to
higher demands for quantity, if both are complements. However, if the costs asso-
ciated with the quality increase are not represented in the price-cap undersupply of quality occurs. Subsequently, assuming quantity and quality being complements, “rate-of-return regulation may have attractive features when quality is a variable” (Spence, 1975). Further efforts have concentrated on verifiability issues of quality (e.g. Lewis and Sappington, 1991, 1992). Not before the spread of incentive regulation in practice (s. below) was the Vogelsang-Finsinger Mechanism (Vogelsang and Finsinger, 1979), which shows the cost adjustment process for price-caps, extended by a quality component and demonstrated that prices which reflect quality still converge towards the second-best solution (de Fraja, 2008).

What holds true in theory can also be observed in practice. With the switch from RoR, which was the dominant regulatory regime at the beginning of the millennium, to incentive based instruments, most of the European countries introduced an quality related incentive component (Schmidtthaler et al., 2015). Schmidtthaler et al. (2015) suggests, that this is due to “suboptimal outcomes from certain incentive regulation schemes”. Obviously, the statement of Ajodhia and Hakvoort (2005), “that at some point the advantages of stricter price regulation – coming from an enhancement of efficiency levels – will not outweigh the additional regulatory costs of setting in place adequate quality regulation” showed to be true. The empirical observable types of performance based quality regulation are summarized by Sappington (2005) or Ajodhia and Hakvoort (2005), for Germany a survey is provided by Diekmann et al. (2007): Either the regulator introduces minimum quality standards or quality bonuses and penalties. Besides of these regulatory options, redesigning the market structure is another option. Another review is provided by Elliott (2006), who additionally mentions ”peer pressure” due to publication of performance indicators as one incentive for monopolists to invest in quality. To measure supply quality in electricity networks, various indicators have been developed. The most commonly used indicators are the system average interruption frequency index (SAIFI), the system average interruption duration index (SAIDI), the customer average interruption duration index (CAIDI), the energy not supplied (ENS) or cost of energy not supplied (CENS) (Ajodhia and Hakvoort, 2005; Diekmann et al., 2007).

In general it can be assumed that quality incentives increase the reliability

Besanko et al. (1987, 1988), however, derives different conclusions when considering heterogeneous consumer preferences. In that case and assuming heterogenous goods, quality provision may be welfare-optimal for consumers with low-quality preferences (s. also Sappington, 2005).
of the electricity grid and that incentive regulation containing explicit quality incentives is superior to RoR as demonstrated by Schmidthaler et al. (2015). Also Ter-Martirosyan and Kwoka (2010) underlines the importance of the inclusion of quality incentives into incentive regulation to decrease outage length. However, some small scale studies derived different conclusions, indicating no effects on quality by the introduction of incentive regulations Elliott (2006). The empirical effects of quality incentives are also equivocal in regard to the trade-off between efficiency gains due to incentive regulation and capital cost increases due to quality incentives. Giannakis et al. (2005) shows, that for cost-based and incentive based regulation monopolists face trade-offs between capital costs (CAPEX) and operating costs (OPEX). A result which could not be observed in an empirical study on Norway Growitsch et al. (2010).

From this short review one can see, that mostly regulatory differences are assumed to be accountable for different quality levels. The only other reasons named are structural differences (Schmidthaler et al., 2015) or weather factors (Yu et al., 2009). However, it might be that path-dependencies are responsible for different quality levels - causing "lock-ins" at low or high quality levels as observed in environmental systems. This subject is did not receive much of attention yet. Therefore, this paper will concentrate on the conditions for the occurrence of path-dependencies from a theoretical perspective.

From a methodological point of view, most of the models named above, which include quality, are either static or in discrete time. Spence (1975) and Sheshinski (1976) deduce their results in a static framework. The same is true for the verifiability literature such as Lewis and Sappington (1991) and Lewis and Sappington (1992). Tangerås (2009) analyses quality provision under yardstick competition in a static model. For intertemporal effects discrete models have been developed by de Fraja (2008) for analysis of an quality adjusted Vogelsang-Finshinger Mechanism, by Currier (2007) taking into account quality-corrected price caps and by Auray et al. (2011) dealing with different contracting schemes. Weber et al. (2010) developed a probabilistic model for the study of optimal replacement strategies in the face of quality penalties. Dynamic and continuous models are only available for evaluating effects on quantity provided (Fellows, 2015; Biglaiser and Riordan, 2000; Niho and Musacchio, 1983; El-Hodiri and Takayama, 1981). The only exception is Besanko et al. (1987, 1988), who provides a dynamic model in continuous time, but does focus on heterogeneous goods and preferences and does not include cost based or incentive instruments.
The specifications of the models are shown in Table 1. What is so far missing, is a dynamic continuous model taking into account effects on quality and quantity, while including incentive and cost-based regulation approaches.

<table>
<thead>
<tr>
<th>Source</th>
<th>Regulation type</th>
<th>Model type</th>
<th>Investment type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Besanko et al. (1987)</td>
<td>Quality only</td>
<td>Dynamic continuous</td>
<td>Quality</td>
</tr>
<tr>
<td>Besanko et al. (1988)</td>
<td>Quality only</td>
<td>Dynamic continuous</td>
<td>Quality</td>
</tr>
<tr>
<td>Niho and Musacchio (1983)</td>
<td>Cost based</td>
<td>Dynamic continuous</td>
<td>Quantity</td>
</tr>
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<td>Biglaiser and Riordan (2000)</td>
<td>Incentive and cost based</td>
<td>Dynamic continuous</td>
<td>Quantity</td>
</tr>
<tr>
<td>Fellows (2015)</td>
<td>Cost based</td>
<td>Dynamic continuous</td>
<td>Quantity</td>
</tr>
<tr>
<td>Vogelsang and Finsinger (1979)</td>
<td>Incentive</td>
<td>Dynamic discrete</td>
<td>Quantity</td>
</tr>
<tr>
<td>Currier (2007)</td>
<td>Incentive</td>
<td>Dynamic discrete</td>
<td>Quantity</td>
</tr>
<tr>
<td>de Fraja (2008)</td>
<td>Incentive</td>
<td>Dynamic discrete</td>
<td>Quantity</td>
</tr>
<tr>
<td>Auray et al. (2011)</td>
<td>Quality only</td>
<td>Dynamic discrete</td>
<td>Quality</td>
</tr>
<tr>
<td>Schill et al. (2015)</td>
<td>Incentive and cost based</td>
<td>Dynamic discrete</td>
<td>Quantity</td>
</tr>
<tr>
<td>Schober and Weber (2015)</td>
<td>Incentive</td>
<td>Dynamic discrete</td>
<td>Quantity</td>
</tr>
<tr>
<td>Weber et al. (2010)</td>
<td>Incentive</td>
<td>probabilistic</td>
<td>Quality</td>
</tr>
<tr>
<td>Spence (1975)</td>
<td>Cost based</td>
<td>Static</td>
<td>Quantity and quality</td>
</tr>
<tr>
<td>Sheshinski (1976)</td>
<td>Incentive</td>
<td>Static</td>
<td>Quantity and quality</td>
</tr>
<tr>
<td>Lewis and Sappington (1991)</td>
<td>Quality only</td>
<td>Static</td>
<td>Quality</td>
</tr>
<tr>
<td>Tangeras (2009)</td>
<td>Incentive</td>
<td>Static</td>
<td>Quantity and quality</td>
</tr>
<tr>
<td>Averch and Johnson (1962)</td>
<td>Cost based</td>
<td>Static</td>
<td>Quantity</td>
</tr>
</tbody>
</table>

Table 1: Overview of models on quality and quantity regulation in regard to type of regulation analyzed, its time frame and the kinds of investments taken into account.
3 The Model and Scenarios for Analysis

I study a situation, where a monopolist supplies a single service with fixed capacity $\bar{x}$ and a variable quality $q(t)$ at time $t$ to a grid user, who demands capacity. To model the behavior of quality investments we will have to theoretically reflect on the meaning of quality in general. Additionally, since we assume that capacity is constant, we have to show the independence of quality and capacity necessary to isolate quality dynamics. These questions are answered in the first theoretical part of this section. In the second part, we will introduce these results into a simple intertemporal optimization model that allows to study quality investments in continuous time. Using this model four different scenarios will be derived. These contain the behavior of the social planner, the unregulated monopolist and the regulated monopolist and serve as ground for the model evaluation in the proximate section.

3.1 Theoretical considerations on modeling quality dynamics and path dependencies

To simplify the theoretical argumentation on how quality investments in energy grids can be modeled, we will first abstract from energy grids and regulation and focus on inherent capital dynamics in general. We do so, because of two assumptions. Firstly, quality $q$ of capacity $\bar{x}$ is assumed to be related to supply quality, so changes in the quality of capacity such as wear and tear effect the quality of supply. Secondly, in the case of electricity infrastructure capacity is assumed to be proportional to the capital stock $K$ providing this capacity. Consequently, increases in capital increase the amount of capacity. By this capital-capacity-quality link, it becomes obvious that capital plays a major role both in regard to quality and capacity. Later on, these general considerations will be applied to the specific context of energy regulation.

3.1.1 On the relation of quality and capital valuation

Thinking about quality in relation to stocks of capital and hence capacity one will not get around the understanding of depreciation. Depreciation is defined as ”the rate of change of asset price with age at a point in time” (Hulten and Wykoff [1981], p.370). Therefore, depreciation (in real terms) takes into account effects by wear and tear on the one hand and value changes of existing capital assets.
due to technical progress and accompanying relative efficiency losses (obsolescence) on the other ([Hulten and Wykoff] 1996 p. 19). Both effects determine the quality change of capital assets, measured by economic depreciation. Usually, it is feasible to assume that overall capital depreciation is constant ([Albonico et al.] 2014 p. 274). However, some studies as [McGrattan and Schmitz] (1999) point at a high volatility of depreciation, which is not well understood yet. This is why another strand of literature assumes that maintenance efforts and capital utilization determine endogenous depreciation rates (see for e.g. the literature mentioned in [Angelopoulos and Kalyvitis] (2012)). In this case, the equation of motion for capital in continuous time has the form

$$\dot{K} = I - \delta K (v/K) K,$$  

(1)

whereas $K$ is the value of capital, $I$ are gross investments in capital extensions ("new" investments) and $v$ accounts for maintenance efforts. Since depreciation $\delta K$ represents quality changes $\dot{q}$, it is feasible to write

$$\dot{K} = I - \dot{q}.$$  

(2)

Consequently, for analyzing quality changes and keeping capital constant ($\dot{K} = 0$) we have to assume that gross investments in expansion $I$ are equal to $\dot{q}(M/K)$. However, since we are not interested in expansion investments, $I$ equals zero so $\dot{q}(M/K)$ must equal zero, which according to the literature on endogenous depreciation is the case when $M \to \infty$ ([Angelopoulos and Kalyvitis] 2012). It becomes obvious that it makes little sense to study quality behavior under these conditions, as quality changes are inherently connected to the valuation of capital and therefore the condition that capacity (as a measure for the capital stock) is constant can not be fulfilled.

3.1.2 Modeling quality of capital

Hence, to study quality changes independently of capital valuation, we have to make some assumptions. On the one hand, we have to assume that $K$ does not represent the value of capital but the amount of capital, like the number of machines or the installed electricity transfer capacity, which we will denote as $\bar{x}$

\footnote{Here we abstract from technical progress as the other component of depreciation mentioned above, because it is not relevant for considerations on quality. Instead we assume a situation without technical progress.}
throughout the remainder of this paper. In doing so, we can say that quality alone determines the value of capital and has no influence on the amount of capacity. This is different for the reversed case, as e.g., high-quality capacity additions would influence aggregate quality levels. But since we avoid looking at changes in capacity due to expansion investments, this influence can be neglected. To my knowledge the only dynamic model that includes quality and its time related changes like depreciation is provided by [Auray et al. (2011)]. Other models, e.g. in the context of regulation like Currier (2007) and de Fraja (2008), concentrate on dynamics of price caps instead of quality and corresponding mechanisms or on quality verifiability Lewis and Sappington (1991, 1992). [Auray et al. (2011)] uses an approach similar to quantity modeling where quality changes dependent on maintenance efforts $v$ and quality depreciation:\footnote{In doing, so we assume that quality also represents the subjective value put on capital by the capital owner due to its capability of producing goods. Hence, capital with quality zero is not able to produce nothing.}

$$q_{t+1} = \delta q_t + v_t \quad \text{with } \delta \in [0, 1].$$  \hspace{1cm} (3)

As we aim at modeling quality in continuous time Eq. 3 is corresponding to

$$\dot{q}(t) = v(t) - \delta q(t).$$  \hspace{1cm} (4)

Even if this approach seems reasonable, [Auray et al. (2011)] does not provide any references on the links between quality changes, depreciation and capital valuation, which is why we will shortly try to justify Eq. 4.

As outlined above, quality changes are related to value changes in the stock of capital due to wear and tear and in special cases also maintenance if we abstract from expansion investments. In the standard formulation for capital dynamics without maintenance the degree of change in the value of capital depends on the present value of capital. The degree of change decreases when the capital value decreases, whereby depreciation becomes a relative measure ($\delta^q = -\frac{\dot{K}}{K}$). This is represented by the second term $-\delta^K q$ in Eq. 4 which also yields the condition that $\dot{q} > 0$ for all $q > 0$. Note that $\delta^q$ does not necessarily equal $\delta^K$ as $\delta^K K = f(\dot{q})$ according to Eq. 1 and 2. Furthermore, by considering endogenous effects on depreciation, maintenance $v$ decreases the degree of change $\dot{q}$ (An-
gelopoulou and Kalyvitis (2012): For \( v \to \infty \) depreciation converges towards zero (\( \delta(v/K) \to 0 \)). It is important to note, that this assumption is not in line with Eq. 4. In the equation used here \( \dot{q} \) can be positive, whereas depreciation is by definition associated with negative effects on quality. However, since it is reasonable to assume that higher maintenance efforts may also absolutely increase quality beyond the depreciation threshold, we can justify the first term of Eq. 4. Take for example a replacement of certain parts of capital by spare parts with higher quality than the original parts due to material differences to picture this effect. To sum up, Eq. 4 provides a coherent representation of quality changes in capital by including all effects mentioned in the endogenous depreciation literature (maintenance, wear and tear) and adds the possibility of quality increases.

3.1.3 Necessary conditions for path dependencies to occur

Using Equation 4 along with further cost equations, we can elaborate the conditions that have to be fulfilled for path dependencies to occur. Let \( v(t) \) be the firm’s investments in quality. Quick investment is costly or, put the other way, there are some adjustment costs, so that costs of capital \( C(v) \) increase in investments \( C_v(v) \geq 0 \) with increasing cost to scale \( C_{vv} > 0 \). This formulation is widely used in the literature see e.g. (El-Hodiri and Takayama, 1981, p.31) or (Lewis and Sappington, 1991, p.372). The stock of quality gives rise to operational cost, \( O(q) \), which increase in quality \( O_q(q) \geq 0 \). Furthermore, assume a natural monopoly situation where prices can be set by the firm, whereby demand is determined by a inverse demand function \( p(q) \) with \( p_q > 0 \), adopted from Spence (1975) and Sheshinski (1976) and adjusted for constant capacity \( \bar{x} \). Assuming the equation of motion for quality is given by Eq. 4 the monopolist maximizes her profits by solving the optimization problem

\[
\max_{v(t)} \int_0^\infty (p(q) - O(q) - C(v)) e^{-\rho t} dt
\]

s.t. \( \dot{q}(t) = v(t) - \delta(q)q(t) \).

By using \( \delta(q) \) we account for possible effects of endogenous quality depreciation in the occurrence of path dependencies. From Equation 5 we can derive a current

\footnote{Here and throughout the text subscripts on cost functions and Hamiltonians denote derivatives.}

\footnote{For better readability we omit the usage of the upper index for the depreciation rate, so that \( \delta = \delta^q \) for the remainder of this paper.}
value Hamiltonian of the form

\[ H = p(q(t)) - C(q(t)) - D(v(t)) + \lambda \dot{q}(t), \quad (6) \]
\[ H = p(q(t)) - C(q(t)) - D(v(t)) + \lambda(v(t) - \delta(q(t))q(t)). \quad (7) \]

In the following I will forgo the explicit notation of \( t \). From the hamiltonian the following conditions for the optimal investment path, which is the stationary path, can be obtained:

\[ H_v = C_v + \lambda = 0, \quad (8) \]
\[ H_q = p_q - O_q + \lambda(\delta_q q + \delta) = r\lambda - \dot{\lambda}. \quad (9) \]

From (8) we can derive equations for \( \lambda \) and \( \dot{v}(t) \)

\[ \lambda = -C_v, \quad (10) \]
\[ \dot{\lambda} = -C_v \dot{v}, \quad (11) \]
\[ \dot{v} = -\frac{\dot{\lambda}}{C_v}. \quad (12) \]

From (9) we can derive equations \( \ddot{\lambda} \)

\[ \dot{\lambda} = \lambda(r + \delta_q + \delta) - p_q + O_q. \quad (13) \]

Therefore, we can obtain the equation for the \( v \)-nullcline by substituing (10) and (13) into the right side of (12) = 0

\[ \dot{v} = 0 = C_v(r + \delta_q q + \delta) + p_q - O_q = F(q, v). \quad (14) \]

Using Equation (14) it is possible to analyze the model for the occurrence of path dependencies (PD). One necessary conditions for PD is the occurrence of at least three multiple steady states within the domain of interest. Steady states are at the cross-sections of the nullclines (\( \dot{q} = 0 \) and \( \dot{v} = 0 \)) of a system of ordinary differential equations (ODEs). In this case, there is no change in either dimension of the system. However, multiple steady states are only possible if there are changes in the slope of the nullclines, otherwise the nullclines will only cross once. The simplest way to investigate this change in slope is by investigating the
change in sign of the slope. If we find reasonable conditions for the occurrence of changes in the sign of the slope, these might indicate PD, even if it is not an sufficient condition. In general, for any implicit function $F(q,v)$ the slope of the function is given by $\frac{dv}{dq} = -\frac{F}{F_q}$. So we calculate $F_q$ and $F_v$ to analyze for a change in sign and assume that $p_{qq} \neq 0$

$$F_v = C_{vv}$$  \hfill (15)

$$F_q = \frac{\delta_{qq} \cdot q}{(+)} + 2 \frac{\delta_q}{(-)} + p_{qq} - O_{qq}$$  \hfill (16)

For a change in sign to occur the function must have a non trivial root. As we assume $C_{vv} > 0$, this is not possible for Eq. 15. On the other hand, Eq. 16 may have a root if at least one term is positive and one is negative and one of them is not constant. If we look at conditions for which exactly one term differs in sign from the other terms, this yields the conditions printed in Table 2.

<table>
<thead>
<tr>
<th>Case</th>
<th>necessary cond.</th>
<th>assumptions for necessity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$O_{qq} &lt; 0$</td>
<td>$\delta_q = 0$ and $\frac{Q_{qq}}{p_{qq}} \neq \text{const.}$ and $(O_{qq} \neq \text{const. or } p_{qq} \neq \text{const.})$</td>
</tr>
<tr>
<td>2</td>
<td>$O_{qq} &lt; 0$</td>
<td>$\delta_q &lt; 0, \delta_{qq} &lt; 0, O_{qq} \neq \text{const.}, \exists O_{qq} \rightarrow</td>
</tr>
<tr>
<td>3</td>
<td>$\delta_{qq} &gt; 0$</td>
<td>$O_{qq} &gt; 0, \delta_q &lt; 0, \exists \delta_{qq} \rightarrow</td>
</tr>
<tr>
<td>4</td>
<td>$\delta_q &gt; 0$</td>
<td>$O_{qq} &gt; 0, \delta_{qq} &lt; 0, \exists \delta_q \rightarrow</td>
</tr>
</tbody>
</table>

Table 2: Necessary conditions for a change in sign in the slope of $\dot{v} = 0$

By further reasoning we can exclude some of these cases, as these conditions only indicate the occurrence of PD. In case one the q-Nullcline is a straight line with slope $\delta$. Consequently, to observe more than one steady state at least two changes in the slope of the v-Nullcline are needed. This is only possible, if the slope of either $p_{qq} \text{ or } Q_{qq}$ has a change in sign, which, however, leads to a cost or price function, which is not monotonically decreasing/increasing and thus does not fulfill standard economic conditions. Case two can be excluded, since no function exists such that $\delta_{qq} < 0$ and $\delta_q < 0$ if $\delta$ must lie within the domain $[0, 1]$. So the only other options are case three and four, which also work for $O_{qq} < 0$.

In both cases depreciation is dependent on quality. To conclude, this indicates that the occurrence of PD depends on the endogeneity of depreciation. This result supports the results from Schober and Weber (2015); Fellows (2015); Biglaiser and Riordan (2000) who show that assumptions about the depreciation rate have an high influence on model outcomes.
3.1.4 Endogenous quality depreciation

In line with the argument of Section 3.1.3 we will investigate whether endogenous quality depreciation is reasonable to assume. However, regarding endogenous depreciation of quality $\delta$ there is no literature available, as depreciation literature always deals with capital valuation. For the development of a coherent argument, we will thus provide some empirical intuition about possible endogenous effects on quality depreciation $\delta$. To do so, let us now turn on the issue of quality of supply in electricity grids. We assume that the quality of the capital stock itself is correlated to the security of supply. For instance low quality power poles might be more prone to extreme weather events or more transformers may increase the value of the grid and grid stability, but not increase capacity. Therefore, here and throughout the paper we will use grid reliability measures for the valuation of quality such as the ”system average interruption duration index” (SAIDI), the ”customer average interruption duration index” (CAIDI) or the amount of ”energy not supplied” (ENS) which are common in the literature (Ajodhia and Hakvoort 2005; Diekmann et al. 2007). Note that by this definition higher values of quality are associated with lower values of the applied measures. Using this assumption, it is interesting to note that security of supply itself seems to impact security of supply: Blackouts are associated with high economic costs for grid operators due to damages on capital, like burnt out transformers. This could be observed for example during the 1977 blackout in New York (Corwin and Miles). Consequently, we assume that low quality values have a negative impact on quality by increasing quality depreciation. This argument makes it possible to use a convex and twice differentiable quality depreciation function in the subsequent manner

$$\delta_q = f(q) \quad \text{with } \delta_q < 0 \text{ and } \delta_{qq} > 0,$$

which according to 3.1.3 is a necessary condition for path dependencies to occur.

3.2 Equations and Scenarios for modeling path dependencies in quality investments

Having derived a formula for modeling path dependencies (PD) we now turn to regulation. In this section we will first derive a time dependent function for regulation that includes different regulatory approaches. After we will make
some specific assumptions on the behavior of cost, demand and quality equations that serve as basis for evaluation of the model.

3.2.1 A dynamic approach to model regulation

To investigate, whether regulation influences quality provision and how these results differ between different regulatory regimes and the unregulated case, we use a time dependent regulatory formula, that includes both cost based and incentive regulation approaches. The price which the monopolist can realize per unit of capacity is constrained by a price-cap, \( p_R \) that is endogenously set by the regulator according to the following regulatory formula:

\[
p(t) \leq p_R(t) = \alpha \frac{O(q(t))}{x} + \beta \frac{C(v(t))}{x} + \gamma q(t)
\] (18)

This formula encompasses the three main regulatory approaches in a stylized way. Depending on its parametrization it can be interpreted as pure cost-plus regulation, pure rate-of-return regulation or as incentive regulation with a reward on quality (see Table 3). Even if this equation captures endogenous changes in price-caps due to changes in the cost structure and quality provision, the behavior heavily depends on the endogenously chosen parameters of \( \alpha \), \( \beta \) and \( \gamma \), which are held constant during one run of the model.

<table>
<thead>
<tr>
<th>Regulation</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>cost-plus</td>
<td>( &gt; 1 )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>rate-of-return</td>
<td>0</td>
<td>( &gt; 1 )</td>
<td>0</td>
</tr>
<tr>
<td>incentive with quality</td>
<td>( &lt; 1 )</td>
<td>( &lt; 1 )</td>
<td>( &gt; 0 )</td>
</tr>
</tbody>
</table>

Table 3: Regulatory regimes

This approach assumes that all different regulatory approaches can be perceived as price caps in the long run. For cost based approaches, prices chosen by the monopolist herself, are limited by the allowed rate of return \( \beta - 1 \) or \( \alpha - 1 \) on capital \( C \) or operating costs \( O \). In this case there is a upper limit to the realized price due to the limitation of profits by the regulator. For incentive regulation, the formula captures the average price adjustment and average costs during one regulatory period and provides incentives for cost reduction, as returns on costs represented by \( \beta \) and \( \alpha \) are below one. However, to take into account cost coverage in the incentive case, the formula includes an specific quality element. Investments which are increasing quality levels increase the price cap and vice
versa. Note, that Equation [18] is binding so \( p = p_R \), if the monopolist can realize the prices-cap limit. For this to happen, prices for a certain capacity level determined from an inverse demand function must exceed the level of the price-cap. We will get into this later on. It is important to mention, that we assume ex ante or ex post verifiable quality levels that are observable by the regulator. Also the regulator has full information about past realized costs associated with particular quantity levels.

3.2.2 Equations used in the model

To model the behavior numerically, we assume certain functions that represent demand, utility, endogenous quality depreciation and cost. For better understanding we choose the functions in the most simple way. Long-term demand for quality and capacity is determined by an inverse demand function of the form

\[
p(q(t)) = a \frac{q(t)}{\bar{x} + 1},
\]

in which \( a \) reflects the willingness to pay. This function is linear in quality and convex in capacity. If we assume \( a > 0 \) this yields the marginal consumer’s valuation of capacity changes \( p_x < 0 \) and quality changes \( p_q > 0 \) and the cross price elasticity \( p_{\bar{x}q} < 0 \). These properties are in line with the assumptions of Section 3.1.3 and represent a standard assumption used in the quality regulation literature (Spence, 1975; Sheshinski, 1976). Therefore, demand increases in quality with constant marginal returns on quality but decreases in quantity. As the sign of the cross-price elasticity \( p_{\bar{x}q} = -\frac{a}{\bar{x}^2} \) is fully determined by the sign of \( a \), assuming \( a > 0 \) means that capacity \( \bar{x} \) and quality \( q \) are complements: Decreases in one variable can be offset by increases of the other variable. This seems reasonable for the case of electricity grids as the height of the price charged for a particular level of capacity increases with quality. However, as capacity is held constant in this model this can also be interpreted as an exogenous factor on the willingness to pay, as higher levels for capacity decrease the height of charged prices for a particular level of quality. The addition by one in the denominator \((\bar{x} + 1)\) guarantees the definition of Eq. [19] for all \( \bar{x} \geq 0 \).

Correspondingly, the utility is given by the area under the demand function, for marginal utility to equal prices \( U_x = p \). To obtain this relation, Equation [19] is integrated by what is ”consumed”, which is capacity according to Spence (1975) and Sheshinski (1976):
\[ U(q(t)) = \int_{0}^{\bar{x}} a \frac{q(t)}{\zeta + 1} d\zeta = aq(t) \ln(\bar{x} + 1). \]  

\((20)\)

For operating costs of the monopolist we assume

\[ O(q) = o \cdot q \cdot \bar{x}, \]  

\((21)\)

whereby operating costs linearly depend on quality and capacity with marginal costs \(o\). This assumption can be justified as certain measures targeting the increase of quality lead to higher operational cost, such as additional backup transformers. Furthermore, according to the assumptions derived in Section 3.1.3 capital costs are given by the twice differentiable convex function

\[ C(q) = c \cdot v^2 \]  

\((22)\)

with \(C_v > 0\) and \(C_{vv} > 0\) due to adjustment costs. This equation can also be interpreted as diminishing returns to capital investments in quality. Using these cost functions we can simplify the regulatory formula given in Equation \((18)\). With \(\phi = \alpha o + \gamma\) the regulatory formula in Equation \((18)\) becomes

\[ p(t) \leq p_{R}(t) = \phi q(t) + \beta \frac{c v(t)^2}{\bar{x}}. \]  

\((23)\)

This will simplify the evaluation of results due to a reduction in parameters. However, in doing so the specification does not make a difference between cost-plus and incentive regulation anymore. Still, this equation captures different incentives for quality provision and returns on capital costs.

Lastly, endogenous depreciation has to be defined on the condition that \(q_{q} > 0\) and \(q_{qq} > 0\) (see Eq. \((17)\)). These dynamics are captured by a parabolic function of the form

\[ \delta(q(t)) = \bar{\delta} \left( \frac{(1 - s)(q(t)^2 - 2q(t)q_{\text{max}})}{q_{\text{max}}^2} + 1 \right) \]  

\((24)\)

which is plotted in Figure \((1a)\) for different values of \(s\). \(\bar{\delta}\) represents the maximum depreciation rate at \(q = 0\), \(s\) is a parameter for the slope of the parabola. Assuming \(s = 1\) yields \(\delta(q_{\text{max}}) = \bar{\delta}\) which represents the standard situation of constant depreciation. For \(s = 0\) it follows that \(\delta(q_{\text{max}}) = 0\) and there will be no depreciation as soon as \(q_{\text{max}}\) is reached: I assume, that there is a maximum quality level
$q_{\text{max}}$, so $q(t) \in V_q = [0, q_{\text{max}}]$ holds, $\delta \in [0, 1]$ is fulfilled and the function is monotonically decreasing and convex for $q(t) \in V_q$. In reality, quality in terms of the proposed measures such as SAIDI has a fixed maximum, so $q_{\text{max}}$ can be selected in such a way so electricity will be granted for the whole year.\footnote{In what follows we omit the explicit notation of $t$, so $q = q(t)$ and $v = v(t)$}

![Diagram](image1.png)

\textbf{(a) Depreciation equilibrium} \hspace{1cm} \textbf{(b) q-Nullclines}

Figure 1: Plot a: Depreciation for different values of $s$ with $q_{\text{max}} = 100, \bar{\delta} = 0.2$. Plot b: Associated q-Nullcline ($\dot{q} = 0$) for different depreciation functions with $q_{\text{max}} = 100, \bar{\delta} = 0.2$

To understand the behavior of quality, the q-Nullcline for Equation 4 is plotted in plot b of Figure 1. The nullcline can be obtained by setting Equation 4 equal to zero ($\dot{q} = 0$). For any pair $q$ and $v$ that satisfies this condition, depreciation equals what is maintained and quality is constant. The corresponding function is

\begin{align}
\dot{q}(t) &= v - \delta(q)q = 0, \\
v &= \delta(q)q \quad \text{with Eq. (24)}, \\
\Leftrightarrow v &= \bar{\delta} \left( \frac{(1-s)(q^2 - 2qq_{\text{max}})}{q_{\text{max}}^2} + 1 \right) q. \tag{27}
\end{align}

Note, that for the special case $s = 0$ in Figure 1 two different parts can be identified, left and right of the extremum. The first part (part I) at low levels of $q$ is monotonically increasing and concave until an extremum is reached. Depreciation rates are high, but due to decreasing quality levels necessary investments
in maintenance decrease with decreasing quality. Thus, in this part the behavior is driven by the quality level. On the right side of the extremum (part II) the function becomes monotonically decreasing. So for decreasing quality levels, \( v \) must increase to keep quality constant. This is due to increases in the depreciation rate. In part II the quality change is driven by the change in depreciation. This effect is low at high quality levels, increases with decreasing quality levels until (convex shape of part II) and after an inflection point (concave shape of part II) decreases with decreasing quality until the extremum. Even if these effects decrease with \( s \to \), due to a smaller slope in the depreciation function, they explain the shapes of Equation 27 in Figure 1.

Having defined these functions the behavior of the monopolist and the social planner is determined by the objective function for maximizing utility or profits in the general form of

\[
\max_{v(\cdot)} \int_0^\infty (R(q) - O(q) - C(v)) e^{-\rho t} dt, \tag{28}
\]

\[
\text{s.t. } (25),
\]

where \( \rho \) denotes the discount rate and \( R \) represents the returns taken into consideration. As we are interested in the long-term behavior of the system, we consider an infinite time horizon. For further analysis, we define different scenarios for the objective function. These differ in the definition of the return equation and the parameters at hand.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>label</th>
<th>( R(q) )</th>
<th>Eq.</th>
<th>Parametrization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Planer</td>
<td>SP</td>
<td>( U(q) )</td>
<td>20</td>
<td>( a &gt; 0 )</td>
</tr>
<tr>
<td>Unregulated Monopolist</td>
<td>UR</td>
<td>( p(q) )</td>
<td>19</td>
<td>( a &gt; 0 )</td>
</tr>
<tr>
<td>Regulated Monopolist</td>
<td>R1</td>
<td>( p_R(q) )</td>
<td>23</td>
<td>( \beta = 0, \phi &gt; 0 )</td>
</tr>
<tr>
<td>Regulated Monopolist</td>
<td>R2</td>
<td>( p_R(q) )</td>
<td>23</td>
<td>( 0 &lt; \beta &lt; \bar{\beta}, \phi &gt; 0 )</td>
</tr>
</tbody>
</table>

Table 4: Scenarios for analysis

In the social planer scenario (SP) overall utility is maximized, whereas in the scenario modeling the unregulated monopolist (UR) profits of the monopolist are maximized. In scenarios with regulation (R1 and R2) profits are maximized on the condition that prices are constrained by price caps. R1 includes incentives on quality due to the inclusion of operational costs and overall quality changes, whereas R2 additionally takes incentives for capital costs into account, whereby the limits for \( \beta \) are explained in Section 3.2.5.
3.2.3 Social Planer behavior

For the Social Planer scenario, welfare is maximized with respect to investments for a given capacity as the net present value of utility and cost, given by

\[
\max_{v(t)} \int_0^\infty \left( aq \ln(\bar{x} + 1) - o\bar{x}q - cv^2 \right) e^{-\rho t} dt,
\]

s.t. (25).

The discount rate is denoted by \( \rho \). Overall net utility is dynamically optimized from \( t \) to \( \infty \), whereas future utility has a discounted value at present. Rewriting Equation (28) in Hamiltonian form (see Appendix A.1) and assuming intertemporal dynamic optimization we can deduce the equation of movement for quality investments given by

\[
v(t)_{SP} = \xi(q, v) + q^2_{\text{max}} (o\bar{x} - a \ln (\bar{x} + 1))
\]

with

\[
\xi(q, v) = 2cv \left[ q^2_{\text{max}} (\bar{\delta} + \rho) + \bar{\delta}q(s - 1)(4q_{\text{max}} - 3q) \right].
\]

The first term of Equation (30) accounts for the dynamics of quality investments and quality changes due to depreciation. It contains the partial derivative with respect to \( q \) of the depreciation function in Equation (24), indicating that marginal change in depreciation plays an important role for the behavior. The second term is a shift of the function dependent on parameterization.

3.2.4 Unregulated Monopolist behavior

In the unregulated case, the profit of the monopolist for a given capacity \( \bar{x} \) is maximized with respect to investments \( v \) as the difference of earning and costs

\[
\max_{v(t)} \int_0^\infty \left( p(q) - o\bar{x}q - cv^2 \right) e^{-\rho t} dt,
\]

\[
\Leftrightarrow \max_{v(t)} \int_0^\infty \left( a \frac{q}{\bar{x} + 1} - o\bar{x}q - cv^2 \right) e^{-\rho t} dt
\]

s.t. (25).

I assume, that the monopolist is perfectly informed about the relation of prices and demand expressed by the willingness to pay within the price function.
Again, using intertemporal dynamic optimization we can obtain the equation of movement for investments, which is

\[ \dot{v}_{UR} = \xi(q) + q_{\text{max}}^2 \left( \frac{-a}{(\bar{x} + 1)} + a\bar{x} \right), \]  

(34)

with \( \xi(q,v) \) from Eq. (31).

Interestingly, the first term of the equation is equal to the one in scenario expressed in Equation (30). Thus the only difference is the last term, which is determined by the parameterization of the model.

### 3.2.5 Regulated Monopolist equations

For simplicity we assume that regulation is binding \( (p = p_R) \), which is valid as long as the willingness to pay determined from the inverse demand function exceeds the regulated price. In this case, the regulated monopolist faces the following decision problem:

\[
\max_{v(\cdot)} \int_0^\infty \left( p_R(q) - \alpha\bar{x}q - cv^2 \right) e^{-\rho t} dt, \tag{35}
\]

\[
\Leftrightarrow \max_{v(\cdot)} \int_0^\infty \left( \phi q + \beta \frac{cv^2}{\bar{x}} - \alpha\bar{x}q - cv^2 \right) e^{-\rho t} dt \tag{36}
\]

s.t. \[ (25) \]

Similar to the previous scenarios, inter-temporal dynamic optimization yields the equation of movement for investments

\[ v(t)_R = \xi(q,v) + q_{\text{max}}^2 \frac{\bar{x}}{(\bar{x} - \beta)} \left( a\bar{x} - \phi \right), \]  

(37)

with \( \xi(q,v) \) from Eq. (31).

Also in the regulated scenario the behavior is driven by \( \xi(q,v) \), and the scenario differs from SP and UR solely in the last term. However, in contrast to the other scenarios an additional conditions must be considered here. The problem is only well behaved \( (H_{vv} < 0) \) if

\[ \beta < \bar{x}. \]  

(38)
Otherwise, the last term changes sign, due to $\bar{x} - \beta$ in the denominator of Equation 37, which completely alters the behavior of the system. This is why $\beta$ is restricted to the domain $[0, \bar{x}]$ in Table 4.

4 Model evaluation and scenario analysis

Based on the equations of motion for quality $q$ and investments $v$, this chapter gives insights in the behavior of the model. This section will try to answer two questions. On the one hand, we will elaborate how the stationary optimal investment path and the optimality of different quality levels of the model is defined. On the other, how do different regulatory incentives influence the behavior of the model. In the first part, we will give a mathematical representation of the model and deduce conditions for steady states dependent on the scenarios derived in the previous section. In what follows, the stability of these steady states is analyzed and a graphical representation of the model is given. This serves as basis for the comparison of the optimality of different steady states in regard to consumer and producer surplus. The section concludes with an extension of the scenarios with regulation, assuming that regulation may become non-binding.

4.1 Steady state conditions in general form

To evaluate the model the steady states for each scenario are derived, which determine the optimal values for investments and quality and the according stationary investment path. At the steady state, both $q$ and $v$ are constant. Therefore, $\dot{q} = 0$ and $\dot{v} = 0$ must be fulfilled. As the $v$-Nullcline in all scenarios only differs in the last term $G_i$ that induces a shift of the nullcline, the solution in scenario $i$ is given as follows:

$$
\dot{v}_i = 2cv \left[ q_{\text{max}}^2 (\bar{\delta} + \rho) + \bar{\delta}q(s - 1)(4q_{\text{max}} - 3q) \right] + G_i = 0, \quad (39)
$$

$$
\dot{q} = v - \bar{\delta} \left( \frac{(1 - s)(q^2 - 2q_{\text{max}})}{q_{\text{max}}^2} + 1 \right) q = 0 \quad (40)
$$

with

$$
G_i \in \left\{ q_{\text{max}}^2 (o\bar{x} - a \ln (\bar{x} + 1)), q_{\text{max}}^2 \left( \frac{-a}{(\bar{x} + 1) + o\bar{x}} \right), q_{\text{max}}^2 \frac{\bar{x}}{(\bar{x} - \beta)(o\bar{x} - \phi)} \right\}.
$$

To calculate the steady state we can derive the Equations that determine $v$ and $q$
respectively. The quality level can be obtained by numerically solving the quintic (Polynomial of degree five) Equation

\[
3\delta^2(1-s)^2 (q_i^*)^5 - 7\delta^2(1-s)^2 q_{max}^5 + \delta^2(1-s) \left( 8 - 4s + \frac{\rho}{\delta} \right) (q_i^*)^3 \\
-2(1-s)q_{max} \delta \left( 3 + \frac{\rho}{\delta} \right) (q_i^*)^2 + \delta q_{max}(\delta + \rho) q_i^* + \frac{G_i}{2c} = 0
\]

which is derived in Appendix A.3. Using the value for \( q_i^* \) from Equation 42 we can obtain the steady state value for investments \( v_i^* \) by using the equation for the \( v \)-nullcline from Equation 51

\[
v_i^* = \left[ \frac{\delta \left( (1-s)(q_i^*)^2 - 2q_i^* q_{max} \right) + 1}{q_{max}^2} \right] q_i^*.
\]

(43)

The stability of the fixed points can be analyzed using the Jacobian Matrix for the model evaluated at the fixed point. The Jacobian represents a multidimensional linear approximation of the system at a given point of interest \( i \). As such, it includes the first order partial derivatives of all variables with respect to all variables of the system. Therefore, the eigenvalues \( \lambda_{jk} \) of the Jacobian reflect "compressions" or "stretches" in each dimension. In the former case, all absolute values of eigenvalues are lower than one. In the latter case at least one value is greater than one. Complex eigenvalues represent "rotations". Using this eigenvalues the stability of each fixed point can be analyzed on the condition that the system is hyperbolic and \( Re(\lambda_{jk}) \neq 0 \quad \forall i \). The Jacobian is given by

\[
J = \begin{pmatrix}
\frac{dv}{dv} & \frac{dv}{dq} \\
\frac{dq}{dv} & \frac{dq}{dq}
\end{pmatrix}
\]

(44)

with

\[
\begin{align*}
\frac{dv}{dv} &= 2c \left[ q_{max}^2 (\delta + \rho) + \delta q^s (s - 1)(4q_{max} - 3q^*) \right], \\
\frac{dv}{dq} &= 2cv^* \left[ q_{max}^2 (\delta + \rho) + \delta (s - 1)(4q_{max} - 6q^*) \right], \\
\frac{dq}{dv} &= 1, \\
\frac{dq}{dq} &= -\delta \left( (1-s)(3q^*)^2 - 4q^* q_{max} \right) + 1).
\end{align*}
\]

The corresponding eigenvalues can be calculated numerically for evaluating this
matrix at the fix points derived from Equation [42] and [43]. Note, that the stability of the system is independent of the regulatory parameters but dependent on the values chosen for endogenous depreciation.

As the difference in the steady state values of the scenarios i in Equation [42] and [43] depends solely on the value of $G_i$, regulation may provide the socially optimal quality and investment levels for certain parameter values. By comparing $G_R$ and $G_{SP}$ insights can be obtained on how the regulator has to adjust his regulatory parameters $(\phi, \beta)$ to achieve the long run socially optimal level of quality provided in the social planer case. The resulting dependence is derived in appendix [A.4] and given by

$$G_{SP} = G_R \text{ if } \phi = o\beta + (\bar{x} - \beta) \frac{a \ln (\bar{x} + 1)}{\bar{x}}. \quad (45)$$

**Proposition 1** (Optimal regulation parameters). *Assuming that the regulator can fully observe the costs of the monopolist and the amount of installed capacity and knows the willingness to pay for quality, regulation is socially optimal in the long run if*

1. *in scenario $R1$ ($\beta = 0$) $\phi$ reflects the marginal utility from quality

$$q_{R1} \begin{cases} \geq \cr < \end{cases} q_{SP}^* \iff \phi \begin{cases} \geq \cr < \end{cases} a \ln(\bar{x} + 1) = U_q \quad (46)$$

2. *in scenario $R1$ ($0 < \beta < \bar{x}$) $\phi$ reflects the marginal utility from quality less the quality improvements already incentived by higher investments through $\beta$

$$q_{R2}^* \begin{cases} \geq \cr < \end{cases} q_{SP}^* \iff \phi \begin{cases} \geq \cr < \end{cases} U_q + \beta(o - \frac{U_q}{\bar{x}}) \text{ mit } U_q = a \ln(\bar{x} + 1). \quad (47)$$

Likewise, we are interested whether the unregulated monopolist provides too much or too little quality compared with the social planer case. Since no regulation occurs and $a$ is equal in both scenarios the results solely depend on capacity $\bar{x}$ which yields

$$\dot{v}_{SP}(t) = \dot{v}_{UR}(t) \text{ if } \bar{x} = 0.76. \quad (48)$$
The derivation is performed in Appendix A.4. Thus we can restate a well known result from static analysis Spence (1975), Sheshinski (1976) in a dynamic setting.

**Proposition 2 (Optimality of unregulated behavior).** The monopolist invests too much or too little in quality in the long run depending on

\[
q_{SP}^* \begin{cases} 
\geq & \text{if } \bar{x} \geq 0.76 \\
< & \text{if } \bar{x} < 0.76
\end{cases}
\]

(49)

The interpretation of this non-intuitive result, which depends on a single number, is given by Spence (1975). By comparing the social planner and the unregulated monopolist in regard to quality, the differences in optimality don’t result from inefficiencies in regulation but from difference in the valuation by either the average and the marginal consumer. Therefore, the valuation of the marginal consumer (taken into account by the monopolist) expressed by the inverse demand function in Equation 19 and average consumer (taken into account by the social planner) expressed by the utility function in Equation 20 become identical for \(x = 0.76\).

### 4.2 Stability analysis and model behavior in general

On basis of the results for steady states derived in the previous chapter, time evolution of investments and quality changes can be analyzed to obtain the optimal investment path for the scenario at hand. We first assume \(\bar{x} = 0.76\), and \(o = c = 1\) and \(\beta = 0\). Assuming these values, the equality conditions from Equations 45 and 48 are satisfied, which simplifies the stability analysis of the steady states. So

\[G_{SP} = G_{UR} = G_R \text{ if } \phi = a \ln (\bar{x} + 1).\]

(50)

The resulting nullclines can be calculated in the same manner as in Section 4.1 (see Equation A.74 in Appendix A.3) and are given by

\[
v = -\frac{q_{max}^2 (o\bar{x} - a \ln (\bar{x} + 1))}{2c [q_{max}^2 (\delta + \rho) + \delta q (s - 1) (4q_{max} - 3q)]} \quad (\dot{v} = 0),
\]

(51)

\[
v = \left(\delta \left(1 - s\right) \frac{q_{max}^2 - 2qq_{max}}{q_{max}^2} + 1\right) q \quad (\dot{q} = 0)
\]

(52)

The steady state values can be derived from these conditions using Equation 42.
Based on the system of Equations, stock dynamics of quality and the optimal investment path, which is the stationary path, can be plotted for different values of $a$ and $\phi$. These phase plots are shown in Figure 2 and 3. $q_{max}$ is chosen randomly, as this model serves conceptual purposes and a change in $q_{max}$ only impacts the values of $a$ for which multistability and monostability occurs, but does not alter the model behavior in general. We analyze the sensitivity in regard to $q_{max}$ and the other parameters later on.

Figure 2 a and b show the saddle point stable steady states associated with a low (a) and high (b) quality level. These steady states are located at the point of intersection of the q-nullcline (Eq. 52) and the v-nullcline (Eq. 51). Independently of the initial quality level the grid operator will chose his investments along the green saddle path, that leads to the saddle point. This path is represented by the stable manifold that results in the steady state. Different steady states exists due to different incentives for investment. At high quality levels investment effort must be higher, than at low quality levels to keep quality constant. In plot a incentives for investment are to low, because either the willingness to pay for quality (scenario SP and UR) or incentives by regulation (scenario R) are to low. In the case of high incentives (higher values for $a$ or $\phi$) it becomes rational to invest in higher quality levels, as there is enough return on investments.

However, at a point in between ($a = 2.25$), bistability emerges in the regime of Figure 3. Circles indicates saddle point stable steady state, rectangles unstable focus. In an unstable focus all phase curves move away from the steady state in a circular dynamic, causing oscillation. The indifference curves of profit are given for different profit levels. One can see, that the right saddle point in region III is associated with the highest profit, whereas profit is lowest for the equilibrium located in region I. In contrast to Figure 2, in the regime of Figure 3 three different decision problems may come into play, depending on the initial quality level $q_0 = q(t = 0)$. If $q_0$ lies in the interval of region I, the grid operator will chose her investments such that she reaches the saddle point associated with low quality levels located in area I. As incentives are to low compared to overall efforts for reaching the high quality level, this decision is optimal. Accordingly, for initial conditions associated with region III additional investment efforts to reach higher quality levels are low, compared to the investment efforts at the initial quality level. Due to decreasing marginal returns on quality, an finite steady state is optimal. If this would not be the case it might be rational to invest to

---

*Compare the argumentation derived for the q-nullcline in Section 3.2.2*
infinity. If the initial quality level is located in region II the monopolist faces two optimal investment paths. For the low quality steady state, she maximizes profits/utility by minimizing investments. For the high quality steady state, she maximizes profits/utility by maximizing returns. However, as the high quality level is associated with higher profits/utility it is rational to choose the latter.

Eigenvalues for different values of $a$ are given in Table 5. Each eigenvalue is obtained by evaluating the Jacobian given in Equation 44 at the steady state and are in line with the dynamics presented in Figure 2 and Figure 3. One positive and one negative real part represent a saddle point, complex eigenvalues with a positive real part represent an unstable focus.

<table>
<thead>
<tr>
<th>$a$</th>
<th>$q^*$</th>
<th>$v^*$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>8.830</td>
<td>1.168</td>
<td>$-0.099$</td>
<td>0.149</td>
</tr>
<tr>
<td>2.25</td>
<td>15.164</td>
<td>1.828</td>
<td>$-0.065$</td>
<td>0.115</td>
</tr>
<tr>
<td>2.25</td>
<td>43.574</td>
<td>3.417</td>
<td>$0.025 + 0.048i$</td>
<td>$0.025 - 0.048i$</td>
</tr>
<tr>
<td>2.25</td>
<td>80.945</td>
<td>3.951</td>
<td>$-0.048$</td>
<td>0.097</td>
</tr>
<tr>
<td>2.5</td>
<td>94.972</td>
<td>4.299</td>
<td>$-0.813$</td>
<td>0.1313</td>
</tr>
</tbody>
</table>

Table 5: Stability analysis for the social planer scenario. Eigenvalues for different equilibria shown in Figures 2 and 3.
Figure 3: Stability regime for $q_{\text{max}} = 100, o = 1, c = 1, x = 1, a = 2.25, \bar{\delta} = 0.15, s = 0.3$. The stability of equilibria is indicated by black dots for saddle points or by a rectangle for an unstable focus. Green lines show stable manifolds of the saddle and saddle path. The purple line represents the b-nullcline, the blue line the q-nullcline.

For the multistability regime of Figure 3, the development of the quality stock over time is shown in Figure 4 for different initial values: In Figure 4(a) initial values for $q$ and $v$ lie within region I, in Figure 4(b) within region II and in (c) within region III. One can see, that depending on the initial condition of quality, different saddle points can be reached, if the initial value lies on the stationary path.

Looking at Figure 4, the decision problem for $q_0$ in region II, becomes more clear. Even if the grid operator starts with the same initial quality $q_0 = 46$, as in plot b, she can either invest along the saddle path towards the low or the high quality saddle point depending on the choice of her initial investment level $v_0$. Consequently, it is on the grid operator to decide, which saddle point to access.

To evaluate the conditions for which multistability occurs, we performed a bifurcation analysis for the the system. By variation of one parameter and holding all others constant one can understand how the the stability regime changes dependent on certain parameters. In Figure 5 the resulting steady states for quality
Figure 4: Development of the quality stock over time for different initial conditions with $q_{\text{max}} = 100, \omega = 1, c = 1, x = 1, a = 2.25\bar{x} = 0.15, s = 0.3$. Graph a has initial values within region we of Figure 3, graph b in region II and c in region II.

$q^*$ are plotted over $a = \phi \ln(\bar{x} + 1)$.

In Figure 6 the change of the bifurcation curve dependent on $\beta \in V_{\beta} = [0, \bar{x}]$ is plotted. Higher incentives on investments represented by $\beta$ decrease the domain for which bistability occurs, because investments become more profitable. Only low incentives for quality, represented by $\phi$, are needed to achieve a high quality equilibrium. Due to the restriction that $q \in V_q$ for high values of $\phi$ no steady state exists that satisfies this condition. Otherwise, we would expect that for high values of $\phi$ high quality levels are achieved.

Even if parameters like $\beta$ influence the domain for bistability, the overall behavior is the same and for any value of $\beta \in V_{\beta}$ some $\phi$ exists for which bistability occurs. As outlined in Section 3.1.3 the occurrence of path dependencies that determine multistability, depends on endogenous depreciation, which is represented by the parameters $\bar{\delta}$ and $s$. For $s \to 1$ depreciation is constant and the necessary conditions of Equation 17 do not hold. This is pictured in plot a of
Figure 5: Bifurcation diagram in scenario SP, R and UR over $a$ with $q_{\text{max}} = 100$, $o = 1$, $c = 1$, $\tilde{\delta} = 0.15$, $s = 0.3$

the two-dimensional stability diagram for $s$ and $\phi$ printed in Figure 7. The plot shows the amount of equilibria at different combinations of $\phi$ and $s$. For low values of $s$ mainly two steady states occur, as the high quality steady state is outside of the domain $V_q$. As the second steady state represents the unstable focus, only low quality levels can be achieved. With increasing $s$ the domain for $\phi$ that contains three equilibria increases, being at it maximum around $s = 0.2$. For even higher values of $s$ only one or three steady states exist, as the high quality steady state is always within the domain $V_q$. For $s > 0.4$ bistability vanishes and there for any $\phi$ only one steady state exists.

**Proposition 3** (Occurrence of path-dependencies). *Assuming endogenous depreciation as expressed by a parabola of the form in Equation [24] it is an necessary condition for the occurrence of path dependencies that the slope of the parabola $s$ does not exceed a lower threshold.*

We can conclude, that for nonlinear endogenous depreciation rates multistability may occur for certain parameter values. The initial quality state restricts the investment decision of the grid operator and path dependencies may occur. So, if the grid operator maximizes inter-temporal profits, she is not able to reach all possible steady states. Whether the steady state that is reached is optimal, will be elaborated in the next section.
Figure 6: Bifurcation diagram for $\phi$ dependent on $\beta$ with parameters $\bar{x} = 0.76, q_{max} = 200, \bar{\delta} = 0.15, s = 0.3$.

Figure 7: Bifurcation in scenario R1 for $s$ and $\phi$ with $q_{max} = 100, o = 1, c = 1, \bar{\delta} = 0.15$
4.3 Optimality: Welfare, consumer and producer surplus

Analyzing the optimality of quality levels, different types of optimality must be distinguished. On the one hand, there is long-run optimality, if fixed costs or capital costs are taken into account. This type of optimality is represented by Section 4.1 as the objective functions used in the model always include capital costs $C$. On the other hand, short-run optimality abstracts from capital costs, taking into account operating costs $O$ only. Usually, this type of optimality serves as ground for comparative statics in regard to welfare. Two conditions for long-run optimality have been derived in Proposition 1 and 7. In this section these will be extended by considerations on short-run consumer and producer surplus and welfare and connected to results from the stability analysis of the previous section and.

4.3.1 Socially optimal regulation

In regard to optimality especially the comparison of scenarios R1/R2 with scenario SP is of major interest, because regulation policies should target the social optimum. Consumer surplus $CS$ for the regulated monopolist is calculated by the difference of utility obtained from capacity $\bar{x}$ "consumption" at a certain quality level and utility losses due to prices on capacity expressed by

$$CS_R(q^*, v^*) = \int_{0}^{\bar{x}} \frac{a}{\psi + 1} q^* d\psi - p_R(q^*, v^*)\bar{x},$$

$$= aq^* \ln(\bar{x} + 1) - \phi q^* \bar{x} - \beta cv^2. \quad (53)$$

Accordingly, producer surplus $PS$ is given by

$$PS_R = p_R\bar{x} - \int_{0}^{\bar{x}} o\psi q^* d\psi,$n

$$= \phi q^* \bar{x} + \beta cv^2 - \frac{1}{2} o\bar{x}^2 q^*. \quad (54)$$

**Proposition 4** (Positive consumer and producer surplus). To obtain positive surplus for consumers and producers the following conditions must be fulfilled for $\phi$:

1. **In case of R1** ($\beta = 0$)

$$\frac{1}{2} o\bar{x} < \phi < \frac{U_q}{\bar{x}}. \quad (55)$$
2. In case of R2 ($0 < \beta < \bar{x}$)

\[
\frac{1}{2} \alpha \bar{x} - \frac{\beta c}{\bar{x}} v^{\ast} \delta(q^{\ast}) < \phi < \frac{U_q}{\bar{x}} - \frac{\beta c}{\bar{x}} v^{\ast} \delta(q^{\ast}).
\]  

(56)

Proof. See Appendix A.6.

However, to obtain the long run social optimum $\phi$ must be chosen according to Proposition 1. For scenario R1 this yields $\phi = U_q < \frac{U_q}{\bar{x}}$ which is true for $\bar{x} < 1$ only.

Besides of consumer and producer surplus, short-run welfare $W$ is determined by the sum of consumer and producer surplus and independent of the scenario at hand, which yields

\[
W = \left( a \ln (\bar{x} + 1) - \frac{1}{2} \alpha \bar{x}^{2} \right) q^{\ast}.
\]  

(57)

It becomes obvious that higher quality levels are associated with higher welfare gains in the short-run as expressed by Equation 57 if $U_q = a \ln (\bar{x} + 1) > \frac{1}{2} \alpha \bar{x}^{2}$. However, there is no finite short-run optimum, due to constant marginal returns on quality. Only by assuming $q \in V_q$ the short-run optimum is finite.

**Proposition 5** (short-run welfare optimum). Assuming that the regulator can fully observe the cost function of the monopolist and the amount of installed capacity and knows the willingness to pay for quality, the short-run (SR) welfare optimum is achieved for

\[
q_{SR}^{\ast} = q_{\text{max}} \quad \text{if} \quad \frac{U_q}{\bar{x}} > 0.5 \alpha \bar{x},
\]  

(58)

\[
q_{SR}^{\ast} = 0 \quad \text{if} \quad \frac{U_q}{\bar{x}} < 0.5 \alpha \bar{x}.
\]  

(59)

Proof. See Appendix A.5.

Note, that the Condition 59 is only of theoretical interest and lacks empirical evidence, because in this case quality regulation would not have been an issue to the policy debate. Thus, the regulator faces a trade-off between short-run and long-run optimality. She should aim at stimulating high quality levels $q_{\text{max}}$, if
she is considering short-run optimality. However, these might not be optimal in the long-run, depending on the endogenously given willingness to pay for quality $a$. This becomes even more important in the face of path dependencies.

If high quality levels are always better in the short-run, it might be feasible for the regulator to adjust the regulation parameters in order to avoid the occurrence of path dependencies. Looking at Figure 5 for $\phi = aln(\bar{x} + 1)$ in region B bistability occurs for both the SP and the R scenario. Consequently, the optimal steady state depends on the initial quality level and investments of the regulated monopolist might target a low quality level as in Figure 3 region I. However, as higher quality is always better in the short-run, it would be better to encourage investments in high quality levels. This can be achieved by selecting $\phi$ in the domain of C of Figure 5. In this case path dependencies disappear and the high quality level will always be achieved and the short-run welfare loss is minimized.

Therefore, we can add an important insight to the static results of Spence (1975) and Sheshinski (1976). Even if regulation can always be set to obtain the long-run social optimum, the short-run welfare difference between the long-run and the short-run steady state can be minimized by selecting regulation parameters that are not optimal in the long-run but in the short-run.

In regard to distributional effects between consumers and producers the ratio of $CS$ and $PS$ can be compared. It is defined by

$$\frac{CS}{PS} = \frac{a \ln (\bar{x} + 1) - \phi \bar{x} - \beta c \delta^2(q^*)q^*}{\phi \bar{x} + \beta c \delta^2(q^*)q^* - \frac{1}{2} \alpha \bar{x}^2}.$$  \hspace{1cm} (60)$$

whereby $\delta^2(q^*)q^* = \frac{(w^*)^2}{q^*}$ (see Equation 43).

**Proposition 6** (Distribution of surplus). The change in the ratio $\frac{CS}{PS}$ over $q^*$ is defined by

$$\frac{dCS/PS}{dq^*} = \frac{\beta c(2\delta(q^*)(q^*) + \delta^2(q^*))((0.5\alpha \bar{x}^2 - a \ln (x + 1))}{(\phi \bar{x} + \beta c \delta^2(q^*)q^* - \frac{1}{2} \alpha \bar{x}^2)^2}.$$  \hspace{1cm} (61)$$

1. For scenario R1 ($\beta = 0$) the distribution of surplus is constant and $\frac{dCS/PS}{dq^*} = 0$.

2. For scenario R2 ($\beta \neq 0$)
\[
\frac{dCS/PS}{q^*} > 0 \text{ for } q^* \in V_q \rightarrow |2\delta_q(q^*)(q^*)| < \delta^2(q^*) \quad (62)
\]
\[
\frac{dCS/PS}{q^*} < 0 \text{ for } q^* \in V_q \rightarrow |2\delta_q(q^*)(q^*)| > \delta^2(q^*) \quad (63)
\]

(64)

**Proof.** See Appendix [A.7].

According to Proposition [6] it is not possible to evaluate the share of surplus in general for scenario R2. The share differs dependent on the parameterization of the model and must be calculated separately.

To sum up, it became clear that in general the regulator faces a trade-off between the short-run and long-run optimum and quality might be locked in a low quality state which is undesirable from a short-run perspective. In this case, the selection of long-run sub-optimal regulation parameters might be feasible. Regarding the distribution of surplus between consumers and producers the distribution is constant for all \(q^*\) for \(\beta = 0\) and not constant for \(\beta \neq 0\). Therefore, only incentives on capital costs cause distributive effects.

### 4.3.2 Inefficiencies due to missing regulation

In Proposition [7] we have stated that under- or oversupply of quality depends on the value of capacity \(\bar{x}\). If \(\bar{x} = 0.76\) the resulting steady states for unregulated and social planer behavior are equal (see Figure [5]). However, as capacity \(\bar{x}\) is exogenously determined, for \(\bar{x} \neq 0.76\) different quality levels for the unregulated and regulated monopolist as pictured in Figure [8] may occur. For example if \(\bar{x} = 0.8\) the steady states are always below the social optimum.

Additionally, also the willingness to pay for quality \(a\) is exogenously given. Dependent on the pair \(\bar{x}, a\) different stability regimes might occur for the social planer and the unregulated scenario.

Take for example \(\bar{x} = 0.8\) and \(a = 2.2\), pictured by the dotted line in Figure [8]. In this case, the unregulated scenario has only one low quality steady state, whereby multistability occurs in the social planer scenario as shown in Figure [9].

Even if for \(\bar{x} \neq 0.76\) the quality level for the unregulated monopolist is either strictly below or above the socially optimal level the difference \(|q^*_{UR} - q^*_{SP}|\) may vary dependent on the initial condition for \(q_0\). If \(q_0\) is located in region we of
Figure 8: Bifurcation for the scenario UR and SP for different $\bar{x}$ with $q_{\text{max}} = 100, \bar{\delta} = 0.15, s = 0.3, o = 1, c = 1, \rho = 0.05$

Figure 9a, in both cases the low quality equilibrium is reached, causing a low difference in quality between SP and UR. However, if $q_0$ lies within region III the socially optimal level of quality is high whereas the unregulated monopolist still invests to the low quality level. Accordingly, the difference between SP and UR is much higher.

As according to Equation 57, higher quality is associated with higher welfare levels the short-run welfare loss due to missing regulation is heavily dependent on the initial quality level, similar to the results obtained by comparing SP and R in Section A.5.

**Proposition 7** (Optimality for the unregulated scenario). For $\bar{x} \leq 0.76$ the quality levels of the unregulated monopolist lies strictly above/below the socially optimal level. However, due to path dependencies the absolute value of welfare loss $W = |q_{\text{UR}}^* - q_{\text{SP}}^*|$ might differ dependent on the initial quality $q_0$ and the willingness to pay $a$ that determine the steady states achieved by the social planer and the unregulated monopolist respectively.

### 4.4 Behavior of the regulated monopolist for non binding regulation

For the scenarios with regulation (R1 and R2) we have assumed that $p = p_R$, because the willingness to pay exceeds the regulated price $p(q) > p_R$. However, if the willingness to pay is below the regulated price the monopolist will not be able to realize the regulated price $p_R$. The behavior of the system changes. For
Figure 9: Phase plots for $\bar{x} = 0.8$ with $a = 2.2, q_{\text{max}} = 100, \bar{\delta} = 0.15, s = 0.3, o = 1, c = 1, \rho = 0.05$.

$p < p_R$ it is consistent with the behavior of the unregulated monopolist, as she is constrained by consumer demand for quality instead of a regulated price-cap. This situation is similar to what is expressed by regulatory holidays in the literature ([Müller et al., 2010] [Gans and King, 2004]). To take into account the possibility of non binding regulation the following conditions can be derived using the equations of movement for the regulated (37) and unregulated monopolist (34) respectively:

$$
\dot{v}_{RUR} = \begin{cases} 
\xi(q, v) + q_{\text{max}}^2 (\frac{\bar{x}}{x-\beta}) (\alpha \bar{x} - \phi) & \text{if } p > p_R \\
\xi(q, v) + q_{\text{max}}^2 \left( \frac{-a}{(\bar{x}+1)} + \alpha \bar{x} \right) & \text{if } p < p_R,
\end{cases}
$$

with

$$
\xi(q, v) = 2cv \left[ q_{\text{max}}^2 (\bar{\delta} + \rho) + \bar{\delta} q (s-1)(4q_{\text{max}} - 3q) \right].
$$

To separate between these two cases a separating curve $v_{SC}(q)$ can be derived, defined at $p = p_R$ (Derivation see Appendix A.8) given by the Equation

$$
v_{SC}(q) = \pm \sqrt{\frac{q_{\text{max}}(a - \phi(\bar{x} + 1))}{c\beta(\bar{x} + 1)}},
$$

which is defined in $\mathbb{R}^+$ for $a > \phi(x + 1)$.

**Proposition 8** (Binding regulation). Assuming an inverse demand function ex-
pressed in Equation 19 and regulated prices given by Equation 23, it is a sufficient condition for non binding regulation that \( a < \phi(\bar{x} + 1) \) as in this case \( p < p_R \), \( \forall q \in V_q \) and \( v \in \mathbb{R}^+ \). For \( a > \phi(\bar{x} + 1) \) regulation is

1. binding, if

   \[
   v_{SC}(q) \leq \sqrt{\frac{q \bar{x}(a - \phi(\bar{x} + 1))}{c\beta(\bar{x} + 1)}}
   \]  

   (68)

2. non binding, if

   \[
   v_{SC}(q) > \sqrt{\frac{q \bar{x}(a - \phi(\bar{x} + 1))}{c\beta(\bar{x} + 1)}}
   \]  

   (69)

Proof. See Appendix A.8.

For the case of \( a > \phi(\bar{x} + 1) \) two exemplary phase diagrams of the model are shown in Figure 10 and 11. In the former, the separating curve is above the steady states of the regulated monopolist (\( v \)-nullcline = blue curve). Below the separating curve, the behavior of the system is represented by the differential equations for the regulated monopolist as \( p > p_R \). Even if there is an interception point of the \( v \)-nullcline of the unregulated monopolist (dashed red curve) and the \( q \)-nullcline (black curve), the interception is located on the wrong side of the separating plane and does not represent a steady state as the dynamics don’t drive the system there. Therefore, for most values of \( q \) in Figure 10, a stationary path exists and the monopolist will optimize so as to reach either the high or low equilibrium dependent on the initial conditions for \( q \). Only for small values of \( q \) the behavior is driven by the differential equations for the unregulated monopolist (\( v \)-nullcline = red line). However, since the equations of motion are derived for the optimum in either the regulated or the unregulated scenario, the monopolist does not take into account the change in the system of equations by crossing the separating curve ex ante. Therefore, it can be supposed that the dynamics don’t represent the intertemporal optimum, if the trajectory crosses the separating plane. Thus, the Figures 11 and 10 only show, whether an accessible steady state exists for the dynamics derived for in Equation 65.

In Figure 11 the opposite case is pictured. All steady states are located above the separating curve. In parallel to the preceding argument, the dynamics are driven by the differential equations derived for the unregulated monopolist. For high levels of quality the dynamics are not optimal, as the stationary path crosses the separating curve.
Figure 10: Binding regulation for $\bar{x} = 0.76, q_{\text{max}} = 100, \delta = 0.15, s = 0.3, a = 3.3, \phi = 1.3$.

Figure 11: Non Binding regulation for $\bar{x} = 0.76, q_{\text{max}} = 100, \delta = 0.15, s = 0.3, a = 3, \phi = 1.45$. 
The regulator is interested to select $\phi$ in order to achieve binding regulation. In the following, we analyze the location of the steady states compared to the separating curve with different values of $\phi$ and $a$. The difference between the fixed points and the separating curve is labeled as shown in Figure 12. $\Delta L$ ($\Delta H$) represents the distance of steady states associated with low (high) quality levels. The indices UR and R indicate the v-nullcline used for calculation. For example $\Delta L_{UR}$ is the difference ($\Delta$) of the separating plane to the low quality steady state (L) at the point of intersection between the q-nullcline and the v-nullcline for the unregulated monopolist (UR). It is only defined for values of $a$ for which a low quality steady state exist (compare the bifurcation diagram in Figure 5). The unstable focus is not taken into account, as it is accessible.

Furthermore, another categorical indicator is used that includes the side of the steady state. "R" indicates that the steady state is located on the "Right" side and "W" that it is located on the "Wrong" side. With "right" we refer to the accordance of steady state and the surrounding system dynamics. In the case of multistability, "H" implies that half of the steady states is on the wrong and one half is on the right side, whereby no distinction is made between which steady state is on which side to simplify the categorization.

![Phase Plot](image)

**Figure 12:** WR: Phase Plot for UR and R with $q_{max} = 100$, $o = 1$, $c = 1$, $x = 1$, $\beta = 0.4$, $\phi = 1.45$, $a = 3\delta = 0.15$, $s = 0.3$
So we can obtain the following conditions:

- **Unregulated Monopolist — right:** \( \Delta_{UR} = v_{SC}(q_{UR}^*) - v_{UR}^* < 0 \)
- **Regulated Monopolist — right:** \( \Delta_{R} = v_{SC}(q_{R}^*) - v_{R}^* > 0 \)
- **Unregulated Monopolist — wrong:** \( \Delta_{UR} = v_{SC}(q_{UR}^*) - v_{UR}^* > 0 \)
- **Regulated Monopolist — wrong:** \( \Delta_{R} = v_{SC}(q_{R}^*) - v_{R}^* < 0 \)

Using this categorization, the differences \( \Delta \) are plotted for \( R \) and \( UR \) in the contour plot of Figure 13. The labels indicate the position of the steady states, whereas the first letter represents the regulated and the second the unregulated steady states. So for \( RW \) all steady states of the regulated v-nullcline are below the separating plane ("right) and all steady states for the unregulated v-nullcline are above the separating plane ("wrong") (this is represented in Figure 10). Additionally, the existence limit derived in Proposition 8 is drawn (purple line). Dotted black lines indicate the multistability limits for \( a \) and \( \phi \) associated with the bifurcation diagram (see Figure 5).

This graph illustrates how \( \phi \) must be selected so as to achieve binding regulation. If the willingness to pay for quality \( a \) is exogenously given and low, \( \phi \) must also be low for regulation to be binding. Otherwise the monopolist can not realize the regulated price. With higher values of \( a \) higher values of \( \phi \) can be selected and regulation is still binding. Furthermore, it becomes clear that only few combinations of \( a \) and \( \phi \) exists where all steady states are located on the wrong side (\( WW \)). For most of the cases either regulation is not binding (\( WR \)) or below the separating curve existence limit or binding (\( RW \)).
Figure 13: Distances of equilibria to separating curve for $q_{max} = 100, o = 1, c = 1, \bar{x} = 1, \beta = 0.4, \delta = 0.15, s = 0.3$ with the categorization (R,H,W) described in this section. The separating curve existence limit is the function for $a = f(\phi)$, where the separating plane is positively defined. Multisability limits show the area for $a$ and $\phi$, respectively, where multistability occurs.

Appendix

A.1 Derivation of Hamiltonians

With $x = \bar{x} = const.$ the Hamiltonian (current-value) of the grid operators optimization problem under regulation is
$$H(q(t), v(t), \lambda) = p_R(q(t)) - C(q(t)) - O(v(t)) + \lambda(t)(v(t) - \delta q(t))$$

$$= \phi q(t) + \beta \frac{cv(t)^2}{\bar{x}} - cv(t)^2 - a\bar{x}q(t) + \lambda(t)(v - \delta q(t)),$$

with

$$\delta(q(t)) = \delta \left( \frac{(1 - s)(q(t)^2 - 2q(t)q_{\text{max}})}{q_{\text{max}}^2} + 1 \right) \quad \text{(A.70)}$$

where $\lambda(t)$ is the co-state on the state equation. The associated necessary first order conditions are

$$H_v = 0 = \frac{2\beta v(t)}{\bar{x}} - 2cv(t) + \lambda(t) \quad \text{(A.71)}$$

$$-\dot{\lambda}(t) + \rho \lambda(t) = \phi - a\bar{x} - \lambda(t)(\delta q(t)q + \delta(q)) \quad \text{(A.72)}$$

in addition to $\dot{q}(t) = v(t) - \delta q(t)$ the transversality condition $\lim_{t \to \infty} \lambda(t)q(t)e^{-\rho t} = 0$ and initial conditions for $q_0 = q(t = 0)$ and $v_0 = v(t = 0)$.

The second order condition is given by

$$H_{vv} = \frac{2\beta c}{\bar{x}} - 2b < 0 \quad \text{(A.73)}$$

So the system is well behaved only for $\beta < \bar{x}$.

From Equation A.71 and A.72 we can derive equations for $\lambda(t), \dot{\lambda}(t)$ and $v(t)$, which together with the state equation lead to the system of two differential equations:

$$\dot{q}(t) = v(t) - \delta q(t)$$

$$v(t)_{SP} = 2cv \left[ q_{\text{max}}^2(\delta + \rho) + \delta q(s - 1)(4q_{\text{max}} - 3q) \right] + q_{\text{max}}^2 (a\bar{x} - a \ln (\bar{x} + 1))$$

Procedure is repeated for scenarios UR and R1/R2. Solving this system of equations numerically for the long-term steady state ($\dot{v}(t) = 0$, $\dot{q}(t) = 0$) establishes the optimal quality.
A.2 Derivation of Extrema for $\dot{v}$

From Equations 37, 30 and 34 we know that all nullclines for $\dot{v}$ have the following properties, with $G$ being a constant dependent on the parameterization.

$$v(t) = \xi + G$$  \hspace{1cm} (A.74)

with

$$\xi = 2cv(t) \left[ q_{\text{max}}^2(\bar{\delta} + \rho) + \delta q(t)(s - 1)(4q_{\text{max}} - 3q(t)) \right].$$

From this equation one can derive the position of the extremum of $\dot{v}$ which is visible in Figure 2 or 3.

$$\frac{d\dot{v}}{dq} = \frac{d\xi(q,v)}{dq}$$ \hspace{1cm} (A.75)

$$= 2c\bar{\delta}v(t)(s - 1)(4q_{\text{max}} - 5q(t))$$

For the Extremum we have:

$$\frac{d\dot{v}}{dq} = 0 \Rightarrow q = \frac{2}{3}q_{\text{max}}$$ \hspace{1cm} (A.76)

A.3 General solution for fixed point values

Given Equation [A.74] and the $\dot{q}$-Nullcline

$$v(t) = \left[ \delta \left( \frac{(1 - s)(q(t)^2 - 2q(t)q_{\text{max}})}{q_{\text{max}}^2} + 1 \right) \right] q(t)$$ \hspace{1cm} (A.77)

we can derive the conditions for the fixed points. For $A.74 = 0$ we obtain

$$0 = 2cv(t)f(q) + G$$

with:

$$f(q) = q_{\text{max}}^2(\bar{\delta} + \rho) + \delta q(t)(s - 1)(4q_{\text{max}} - 3q(t))$$

$$\Leftrightarrow v(t) = \frac{-G}{2cf(q)}.$$ \hspace{1cm} (A.78)

Setting Equation [A.78] and [A.77] equal yields

$$\frac{-G}{2cf(q)} = \delta(q)q.$$ \hspace{1cm} (A.79)
Solving Equation A.79 for q and doing some manipulations gives the implicit closed form equation

\[
3 \bar{\delta}^2 (1 - s)^2 q^5 - 7 \bar{\delta}^2 (1 - s)^2 q^4 + \bar{\delta}^2 (1 - s) \left( 8 - 4s + \frac{\rho}{\bar{\delta}} \right) q^3 \\
-2(1 - s)q_{\max} \bar{\delta} \left( 3 + \frac{\rho}{\bar{\delta}} \right) q^2 + \bar{\delta}q_{\max}(\bar{\delta} + \rho)q + \frac{G}{2c} = 0
\]

We can use Equation A.80 to calculate the fixpoint values for q of the different scenarios, depending on the parameters in place. By substituting the solution into Equation A.77 we obtain the fixpoint value for v.

### A.4 Equality conditions for scenarios

Given the following terms that determine the shift for \( \dot{v} \).

\[
G_{SP} = q_{\max}^2 (\bar{o}x - a \ln (\bar{x} + 1)) , \quad (A.81) \\
G_{UR} = q_{\max}^2 \left( \frac{-a}{\bar{x} + 1} + \bar{o}x \right) , \quad (A.82) \\
G_{R} = q_{\max}^2 \frac{\bar{x}}{(\bar{x} - \beta)} (\bar{o}x - \phi) . \quad (A.83)
\]

So \( G_{SP} = G_{R} \) if

\[
q_{\max}^2 (\bar{o}x - a \ln (\bar{x} + 1)) = q_{\max}^2 \frac{\bar{x}}{(\bar{x} - \beta)} (\bar{o}x - \phi) \quad (A.84) \\
\iff \bar{o}x - a \ln (\bar{x} + 1) = \frac{\bar{x}}{(\bar{x} - \beta)} (\bar{o}x - \phi) \quad (A.85) \\
\iff \frac{(\bar{x} - \beta)}{\bar{x}} (\bar{o}x - a \ln (\bar{x} + 1)) - \bar{o}x = -\phi \quad (A.86) \\
\iff \phi = \bar{o}x - ((\bar{x} - \beta)a - (\bar{x} - \beta)\frac{a \ln (\bar{x} + 1)}{\bar{x}}) \quad (A.87) \\
\iff \phi = \bar{o} \beta + \left( \frac{a \ln (\bar{x} + 1)}{\bar{x}} \right) \quad (A.88)
\]

Accordingly, for \( G_{UR} = G_{R} \) we can derive

\[
q_{\max}^2 \left( \frac{-a}{\bar{x} + 1} + \bar{o}x \right) = q_{\max}^2 (\bar{o}x - a \ln (\bar{x} + 1)) , \quad (A.89) \\
\iff \frac{1}{\bar{x} + 1} = \ln (\bar{x} + 1) \quad (A.90) \\
\iff \text{fulfilled if } \bar{x} = \frac{1}{W(1)} - 1 = 0.76 \quad (A.91)
\]
A.5 Short-run welfare optimum

Short-run welfare $W$ is defined by

$$W = (a \ln (\bar{x} + 1) - \frac{1}{2} \alpha \bar{x}^2)q^*.$$  \hfill (A.92)

If $q \in V_q$ the short-run optimum value for $q^*_{SR}$ depends on the sign of $(a \ln (\bar{x} + 1) - \frac{1}{2} \alpha \bar{x}^2)$. Consequently

$$q^*_{SR} = q_{max} \quad \text{if} \quad a > \frac{\alpha \bar{x}^2}{2 \ln (\bar{x} + 1)} \iff \frac{U_q}{\bar{x}} > 0.5 \alpha \bar{x}, \quad (A.93)$$

$$q^*_{SR} = 0 \quad \text{if} \quad a < \frac{\alpha \bar{x}^2}{2 \ln (\bar{x} + 1)} \iff \frac{U_q}{\bar{x}} > 0.5 \alpha \bar{x}. \quad (A.94)$$

A.6 Positive Surplus

In scenario R1 consumer surplus is defined by

$$CS_{R1} = (a \ln (\bar{x} + 1) - \phi \bar{x})q^* = (U_q - \phi \bar{x})q^*, \quad (A.95)$$

$$\Rightarrow CS_{R1} > 0 \quad \text{if} \quad \phi < \frac{U_q}{\bar{x}}. \quad (A.96)$$

Accordingly, producer surplus is defined by

$$PS_{R1} = (\phi \bar{x} - \frac{1}{2} \alpha \bar{x}^2)q^*, \quad (A.97)$$

$$\Rightarrow PS_{R1} > 0 \quad \text{if} \quad \phi > \frac{1}{2} \alpha \bar{x}. \quad (A.98)$$

For scenario R2 consumer surplus is defined by

$$CS_{R1} = (a \ln (\bar{x} + 1) - \phi \bar{x} - \beta c(v^*)^2)q^* = (U_q - \phi \bar{x} - \beta c(v^*)^2)q^* \quad (A.99)$$

$$\Rightarrow CS_{R1} > 0 \quad \text{if} \quad \phi < \frac{U_q}{\bar{x}} - \frac{\beta c (v^*)^2}{q^*} = \frac{U_q}{\bar{x}} - \frac{\beta c}{\bar{x}} v^* \delta(q^*). \quad (A.100)$$

Accordingly, producer surplus is defined by

$$PS_{R1} = (\phi \bar{x} + \beta c(v^*)^2 - \frac{1}{2} \alpha \bar{x}^2)q^*, \quad (A.101)$$

$$\Rightarrow PS_{R1} > 0 \quad \text{if} \quad \phi > \frac{1}{2} \alpha \bar{x} - \frac{\beta c}{\bar{x}} v^* \delta(q^*). \quad (A.102)$$
A.7 Surplus ratio changes by quality

The consumer-producer surplus ratio is defined as

\[
\frac{CS}{PS} = \frac{a \ln (\bar{x} + 1) - \phi \bar{x} - \beta c \delta^2(q^*)q^*}{\phi \bar{x} + \beta c \delta^2(q^*)q^* - \frac{1}{2} a \bar{x}^2}.
\] (A.103)

To analyze how the ratio changes with changes in quality Equation A.103 is derived with respect to \(q^*\). This yields

\[
\frac{dCS}{PS} = \frac{\beta c (2 \delta q(q^*) + \delta^2(q^*)) (0.5 a \bar{x}^2 - a \ln (x + 1))}{(\phi \bar{x} + \beta c \delta^2(q^*)q^* - \frac{1}{2} a \bar{x}^2)^2}.
\] (A.104)

I know from Equation 24 that \(\delta q < 0\). Furthermore, welfare \(W = a \ln (1 + x) - 0.5 a \bar{x}^2 > 0 \Rightarrow 0.5 a \bar{x} - a \ln (1 + x) < 0\). Therefore the numerator is negative if \(|2 \delta q(q^*)(q^*)| > \delta^2(q^*)\) and reverse. The squared denominator is always positive for \(q \in V_q\). Consequently for \(\beta \neq 0\) consumers profit more from higher quality levels than producers if \(|2 \delta q(q^*)(q^*)| < \delta^2(q^*)\). For \(\beta = 0\) the ratio is constant.

A.8 Derivation of separating curve

The separating curve (SC) at \(p = p_R\) is defined by

\[
\frac{aq}{\bar{x} + 1} = \phi q + \frac{\beta c v^2}{\bar{x}},
\]

\(\iff v_{SC}(q) = \pm \sqrt{\frac{q \bar{x} (a - \phi(\bar{x} + 1))}{c \beta (\bar{x} + 1)}}\) (A.105)

\(v_{SC}\) is defined in \(\mathbb{R}^+\) if \(a > \phi(\bar{x} + 1)\). Therefore, two cases can be derived for the inequality condition \(p = p_R\).

\[
p \begin{cases} 
\geq & p_R \\
= & \end{cases} \quad \begin{cases} 
v_{SC}(q) \leq \sqrt{\frac{q \bar{x} (a - \phi(\bar{x} + 1))}{c \beta (\bar{x} + 1)}} & \text{if } a > \phi(\bar{x} + 1) \\
v_{SC}(q) > \sqrt{\frac{q \bar{x} (a - \phi(\bar{x} + 1))}{c \beta (\bar{x} + 1)}} & \text{if } a > \phi(\bar{x} + 1) \\
a < \phi(\bar{x} + 1) & \forall q \in V_q \text{ and } v \in \mathbb{R}^+
\end{cases}\] (A.107)

For \(a < \phi(\bar{x} + 1)\) \(p\) is always lower than \(p_R\) and no binding regulation exists.
### Abbreviations

<table>
<thead>
<tr>
<th>Label</th>
<th>Variable</th>
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<tbody>
<tr>
<td>Capacity of the grid</td>
<td>$\bar{x}$</td>
</tr>
<tr>
<td>Quality of the grid</td>
<td>$q$</td>
</tr>
<tr>
<td>Investments in quality</td>
<td>$v$</td>
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<tr>
<td>Time</td>
<td>$t$</td>
</tr>
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<td>Depreciation rate of quality</td>
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<td>Discount rate</td>
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<td>Inverse demand</td>
<td>$p(q, x)$</td>
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<td>Returns</td>
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<td>Regulation parameter</td>
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<td>Willingness to pay for quality</td>
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<td>Marginal operating costs</td>
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<td>Capital Costs coefficient</td>
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References


