Directed Technical Change and Energy Intensity Dynamics: Structural Change vs. Energy Efficiency*

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Abstract

This paper uses a model with Directed Technical Change to theoretically analyse observable heterogeneous energy intensity developments. Based on the empirical evidence, we decompose changes in aggregate energy intensity into structural changes in the economy (*structural effect*) and within-sector energy efficiency improvements (*efficiency effect*). The relative importance of these effects is determined by energy price growth and sectoral productivities that drive the direction of technical change. When research is directed to the labour-intensive sector, the *structural effect* is the main driver of energy intensity dynamics. In contrast, the *efficiency effect* dominates energy intensity developments, when research is directed to energy-intensive industries. Increasing energy price generally leads to lower energy intensities and temporal energy price shocks might induce a permanent redirection of innovation activities. We calibrate the model to empirical data and simulate energy intensity developments across countries. The results of our very stylised model are largely consistent with empirical evidence.

Keywords: Directed technical change, Energy efficiency, Energy intensity, Structural change.

JEL-Classification: O33, O41, Q43, Q55.

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1. Introduction

The relationship of energy use and economic activity has been a recurring theme in the political and academic debate, particularly since the energy crisis in the 1970s. Main reasons include the high dependency on fossil fuel energy carriers in energy generation – 80.6% in 2014 (IEA, 2015) – and the resulting consequences for the world climate as well as increasing energy prices. A promising way to lower emission levels and meet climate policy targets is reducing energy intensity, i.e. decreasing the input of energy for production of a given output.

Since the energy crisis in the 1970s, numerous studies have analysed the development of energy intensities.[1] Studies covering the period before the energy crisis, i.e. 1950 - 1970, show increasing or constant energy intensities across most of the analysed developed and emerging economies (Casler and Hannon, 1989; Hannesson, 2002; Proops, 1984). For the period after the energy crisis, however, there is strong evidence for substantial reductions in energy intensity in the majority of developed economies (Csereklyei et al., 2016; Greening et al., 1998; Grossi and Mussini, 2017; Liddle, 2012; Mulder and de Groot, 2012; Sun, 1998; Voigt et al., 2014; Wang, 2013).[2] In addition to analysing trends of overall energy intensities across countries, numerous studies use, e.g., index- or structural decomposition analyses to disaggregate energy intensity into its driving forces (Ang, 2004; Löschel et al., 2015; Mendiluce et al., 2010; Mulder, 2015; Sue Wing, 2008).[3] Most studies decompose energy intensity into an *efficiency effect*[4] and a *structural effect*. The former describes energy efficiency improvements within sectors, i.e. reductions in sectoral energy intensities due to e.g. substitution of energy by other factors or energy-saving technological progress. The *structural effect* refers to structural adjustments towards sectors with low energy intensities.

Mulder and de Groot (2012) decompose the development of energy intensities across 50 sectors in 18 OECD countries for the period 1970-2005. The authors find an important contribution of the structural effect for energy intensity reductions (25% in all analysed OECD economies). However, the relative importance of the efficiency effect seems to be stronger.[5] A

[1]A theme related to the energy intensity literature is the so-called rebound effect, which can be decomposed into a direct rebound, an indirect rebound, and an economy wide (or growth) effect (Binswanger, 2001; Brookes, 2004; Greening et al., 2000; Khazzoom, 1980, 1987; Qiu, 2014; Schipper and Grubb, 2000).

[2]Greening et al. (1998) analyse ten developed economies from 1971-1991 and find energy intensity reductions between 37.5% (Norway) and 61.7% (Japan). For a similar period (1973-1990), Sun (1998) finds a reduction of energy intensity of 26.2% across OECD countries. Liddle (2012) and Wang (2013) find similar results using more recent data. In spite of continuous reductions in energy intensities, there is still a high potential for energy efficiency improvements (Velthuijsen, 1993; Worrell et al., 2009).

[3]Ang and Zhang (2000) found 124 studies applied decomposition analyses related to energy-based emissions and energy demand. Only ten years later, the number of studies almost doubled (Su and Ang, 2012).

[4]The efficiency effect is also referred to as technology or (sectoral) intensity effect.

[5]Sun (1998) finds a contribution of the efficiency effect of 75.5% from 1973-1980 that even increased to 90% from 1980-1985 and 92.8% from 1985-1990. Greening et al. (1998) also find that energy efficiency improve-

recent and very comprehensive decomposition analysis was conducted by Voigt et al. (2014). Using the World Input-Output Database (WIOD) covering 34 sectors in 40 countries from 1995-2007, Voigt et al. (2014) show a conspicuous divergence in the importance of the structural and the efficiency effect for energy intensity developments across countries. In around a third of all developed economies energy intensity reductions are primarily caused by a restructuring of the economy towards sectors with low energy intensities (structural effect). In the reminder of all industrial countries, the efficiency effect is primarily responsible for the decline in energy intensity. Overall, the data analyses on energy intensities show the following trends:

i. while energy intensities were constant or increasing in the majority of economies until the early 1970s, they systematically decreased since the energy crisis;

ii. the contributions of energy intensity reductions within industries, e.g. through technological progress, (efficiency effect) and structural change towards less energy-intensive economic activities (structural effect) to energy intensity reduction differ substantially across countries.

In contrast to the extensive empirical literature on energy intensity developments, there is a lack of theoretical approaches to analyse the underlying mechanisms of the trends described above. Recent studies, as Mulder and de Groot (2012) and Voigt et al. (2014), highlight the exploration of the determinants of these developments, including the role of technological change, as directions of future research. Our paper aims to fill this research gap by providing a, to our knowledge, first theoretical analysis of energy intensity dynamics.[6] We analyse how endogenous technical change and energy price affect the direction and magnitude of the structural and the efficiency effect.

For this purpose, we use a theoretical Directed Technical Change (DTC) framework as proposed by Acemoglu (1998, 2002) to analyse the observed trends in energy intensity dynamics. The application of DTC model frameworks to examine the relation of technical change and the use of energy or natural resources is not new. Di Maria and van der Werf (2008) use a two-sector DTC model with an energy- and a labour-intensive sector to analyse the effect of unilateral climate policy on carbon leakage, while Di Maria and Smulders (2004) examine the pollution haven effect. Di Maria and Valente (2008) and Pittel and Bretschger (2010) study whether technical change is research resource-augmenting within DTC model frameworks. André and Smulders (2014) investigate long-run trends in oil dependency by introducing energy input from non-renewable resources into a DTC model setup.

ments within sectors are the main drivers of energy intensity decline.

[6] A recent exception is Cao (2017), who uses a different model framework. A main difference is that the author explicitly models the production of energy. In contrast to Cao (2017), the direction of technical change is determined endogenously in our analysis.

Our analysis mainly builds on the DTC model of Acemoglu et al. (2012). We use a marginally modified version of their model with exhaustible resources, as it ideally serves the purpose of our analysis. Since we want to analyse the effect of energy prices on innovation and energy use, we need a model framework with energy as input factor and endogenous technical change. Furthermore, we require a multi-sectoral setup to explicitly differentiate between structural adjustments between sectors and within-sector energy efficiency improvements. Based on the model, we provide new insights on the effects of energy price growth and endogenous technical change on energy intensity developments. We show how energy price growth and the relative productivity of the labour- and the energy-intensive sector affect the direction of technical change. After decomposing overall energy intensity into efficiency effect and structural effect, we show how the direction and magnitude of both effects is affected by technical change and energy price growth. We find that the efficiency effect dominates the evolution of energy intensity in economies, when research is directed to the energy-intensive sector. When research is directed to the labour-intensive sector, the structural effect is the main driver of energy intensity dynamics. By calibrating the model to empirical data for 26 OECD countries, we illustrate how energy intensity development, driven by these two effects, varies across these countries.

The remainder of the paper is structured as follows. In Section 2 we present the model and characterise the equilibrium. Section 3 contains the main analysis. We decompose energy intensity into structural and efficiency effect and show how both effects are affected by technical change and energy price growth. Section 4 provides simulations to illustrate the model results. In Section 5, we discuss our results and possible extensions of the model. Section 6 concludes.

2. The Model

In this section, we introduce the model framework, which is based on the setup of Acemoglu et al. (2012) and modified in the following ways. The authors model the energy price as function of the resource stock, since they analyse how the depletion of an exhaustible resource might induce a redirection of technical change towards a clean sector due to a continuously increasing price. In contrast, we model an exogenous price for energy and endogenous energy use, as our focus is the analysis of heterogeneous energy intensity dynamics across countries and historical scenarios with different energy price growth rates. Furthermore, we formulate our model in continuous time. This redefinition of the time dimension allows an extension of the model by an analytical decomposition of energy intensity into a structural and an efficiency effect, which we present in Section 3.

2.1. Model Framework

Consider an economy with infinitely-lived households consisting of scientists, entrepreneurs, and workers. Consumer behaviour can be described by a representative household that maximises its utility (U) through consumption, $C(t)$, of the only final product at time t with the utility function

$$U \equiv \int_0^\infty e^{-\rho t} u\left(C(t)\right) dt, \tag{1}$$

where ρ is the rate of time preference. The unique final good ($Y(t)$) is assembled from sectoral outputs of a labour-intensive sector ($Y_l(t)$) and an energy-intensive sector ($Y_e(t)$) according to

$$Y(t) = \left(Y_l(t)^{\frac{\epsilon-1}{\epsilon}} + Y_e(t)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}. \tag{2}$$

Markets for $Y(t)$, $Y_l(t)$, and $Y_e(t)$ are perfectly competitive. The outputs of the labour-intensive and the energy-intensive sector are imperfect substitutes, where ϵ (with $\epsilon > 0$) is the elasticity of substitution between both goods. In the following, the two goods will be referred to as (gross) substitutes when $\epsilon > 1$ and (gross) complements in the case of $\epsilon < 1$. $\epsilon = 1$ is not considered, as in this case the production function converges to the Cobb-Douglas type and hence technical change is neutral to the input factors.

In each sector $j \in \{l, e\}$, labour ($L_j(t)$) and a sector specific set of machines are used for production. Each machine type i in sector j, $x_{ji}(t)$, has an individual productivity $A_{ji}(t)$. The production in the energy-intensive sector additionally requires energy $E(t)$. The production functions of both sectors are:

$$Y_l(t) = L_l(t)^{1-\alpha} \int_0^1 A_{li}(t)^{1-\alpha} x_{li}(t)^{\alpha} di, \tag{3}$$

$$Y_e(t) = E(t)^{\alpha_2} L_e(t)^{1-\alpha} \int_0^1 A_{ei}(t)^{1-\alpha_1} x_{ei}(t)^{\alpha_1} di, \tag{4}$$

with $\alpha = \alpha_1 + \alpha_2$, $\alpha \in (0, 1)$. The aggregate productivity of sector $j \in \{l, e\}$ is defined as

$$A_j(t) \equiv \int_0^1 A_{ji}(t) di. \tag{5}$$

This definition will be used for the subsequent analysis of the direction of research. Labour is assumed to be supplied inelastically. Normalising labour supply to 1, the labour market clearing condition is

$$L_l(t) + L_e(t) \leq 1. \tag{6}$$

Energy $E(t)$ is supplied at per unit costs of $c_E(t)$. With respect to the evolution of energy costs over time, we consider different scenarios that are discussed in Section 3.

Machines are produced with an identical, linear production technology at identical costs of ψ units of the final product and supplied under monopolistic competition. Market clearing for the unique final good implies

$$Y(t) = C(t) + \psi\left(\int_0^1 x_{li}(t)di + \int_0^1 x_{ei}(t)di\right) + c_E(t)E(t). \tag{7}$$

Technological progress is driven by quality improvements of machines. Each machine is owned by an entrepreneur, the measure of entrepreneurs in each sector is normalised to one, respectively. At the same time, scientists (entrants) attempt to enter the market (become an entrepreneur) through innovation. Scientist direct their research at either the labour- or energy-intensive sector. With the probability $\eta_j \in (0, 1)$, the innovation attempt is a success and the scientists is randomly allocated to a specific machine, increases its quality by $\gamma > 0$, receives a patent, and becomes the sole producer of this machine variety. The entrepreneur that used the old version of this machine leaves the market and joins the pool of scientists. Normalising the mass of scientists to one, the market clearing condition for scientists is

$$s_l(t) + s_e(t) \leq 1, \tag{8}$$

with s_{jt} denoting the mass of scientists directing their research at sector $j \in \{l, e\}$. Due to this innovation process, together with (5), the aggregate sector productivity, $A_j(t)$, improves over time according to the following law of motion:

$$\dot{A}_j(t) = s_j(t)\eta_j\gamma A_j(t). \tag{9}$$

2.2. Research Incentives and Directed Technical Change

In this subsection, we define the equilibrium, which is formally derived in Appendix B, and analyse the direction of research.

Definition 1. *An equilibrium is given by prices for sector outputs ($p_j(t)$), machines ($p_{ij}(t)$) and labour ($w_j(t)$), demands for machines ($x_{ji}(t)$), the exogenous energy price ($c_E(t)$), sector outputs ($Y_j(t)$), labour ($L_j(t)$) and energy ($E(t)$) of sector $j = \{e, l\}$, such that at t: $p_{ij}(t), x_{ij}(t)$ maximizes profits of producers of machine i in sector j; $L_e(t), E(t)$ maximizes profits of producers in the energy intensive sector, $L_l(t)$ maximizes profits of producers in the labour intensive sector; $Y_j(t)$ maximizes profits of final good producer; $s_j(t)$ maximizes expected profits of researchers in sector j.*

In order to determine technical change, i.e. the development of productivities in the energy-intensive and the labour-intensive sector, the direction of research has to be examined. The research incentives of scientists are determined by the profitability of research in both sectors, i.e. the expected firm value due to the patented innovation in the respective sector. Following Acemoglu et al. (2012) and Daubanes et al. (2013), a patent for an improved sector specific machine is enforced for the smallest definable unit of time. This assumption simplifies the expected firm value to the profit in t.[7] Combining (B.13), (B.15), and (B.16) with (C.1) yields the relative profitability of research as:

$$\frac{\Pi_l(t)}{\Pi_e(t)} = \kappa \frac{\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}{\eta_e} \frac{A_l(t)^{-\varphi}}{A_e(t)^{-\varphi_1}} \tag{10}$$

with $\kappa \equiv \frac{(1-\alpha)\alpha}{(1-\alpha_1)} \left(\frac{\alpha^{2\alpha}}{\psi^{\alpha_2}\alpha_2^{\alpha_2}}\right)^{\epsilon-1} \alpha_1^{2\alpha_1(1-\epsilon)-1}$, $\varphi \equiv (1-\alpha)(1-\epsilon)$, $\varphi_1 \equiv (1-\alpha_1)(1-\epsilon)$. Relative profitability is a function of time-invariant parameters, the energy price, research efforts in both sectors as well as productivities. The following lemma can be derived from expression (10).

Lemma 1. *In equilibrium, research is directed to the energy-intensive sector only, when* $A_e(t)^{(-\varphi_1)}\eta_e > \kappa\eta_l c_E(t)^{\alpha_2(\epsilon-1)} A_l(t)^{(-\varphi)}$*, to the labour-intensive sector only, when* $A_e(t)^{(-\varphi_1)}\eta_e < \kappa\eta_l c_E(t)^{\alpha_2(\epsilon-1)} A_l(t)^{(-\varphi)}$*, and to both sectors, when* $A_h(t)^{(-\varphi_1)}\eta_e = \kappa\eta_l c_E(t)^{\alpha_2(\epsilon-1)} A_l(t)^{(-\varphi)}$.

Proof. See Appendix C. □

This means that for $\epsilon > 1$, research is directed to the technically more advanced sector whereas for $\epsilon < 1$ the less advanced sector is favoured. In addition to the technological level of both sectors, the exogenous energy price affects research incentives. In general, an increasing energy price increases (decreases) the profitability for innovation in the labour-intensive sector for $\epsilon > 1$ ($\epsilon < 1$). Whether this effect of the energy price ultimately dominates the direct productivity effect depends on the growth rates of the energy price and the technologies. Analysing the growth rate of the relative profit yields the following lemma:

Lemma 2. *i. With moderate growth of the energy price, i.e. the growth rate remains in the band* $-\eta_l\gamma(1-\alpha)/\alpha_2 \leq \gamma_{c_E} \leq \eta_e\gamma(1-\alpha_1)/\alpha_2$*, the direction of technical change is determined by relative productivity that dominates the effect of energy price growth.*

ii. Strong growth of the energy price, i.e. $\gamma_{c_E} > \eta_e\gamma(1-\alpha_1)/\alpha_2$*, will ultimately lead to research in the l- (e-) sector for* $\epsilon > 1$ ($\epsilon < 1$).

[7] A detailed analysis of the direction of technical change with longer (infinite) duration, where the scientist derives monopoly profits until another scientist improves her machine variety and hence replaces her, can be found in Appendix E. Although this approach is more general, this simplification does not affect our further analysis.

iii. Strong negative growth of the energy price, i.e. $\gamma_{c_E} < -\eta_l\gamma(1-\alpha)/\alpha_2$, *will ultimately lead to research in the e- (l-) sector only for* $\epsilon > 1$ ($\epsilon < 1$).

Proof. See Appendix C. □

Finally, we impose the following three assumptions based on Lemma 1, which will be useful for the subsequent analysis.

Assumption 1. $A_e(t)^{(-\varphi_1)}/A_l(t)^{(-\varphi)} < \kappa c_E(t)^{\alpha_2(\epsilon-1)}\eta_l/\eta_e$.

Assumption 1 implies that the l-sector's technological advancement results in research in the l-sector (e-sector) only for $\epsilon > 1$ ($\epsilon < 1$).

Assumption 2. $A_e(t)^{(-\varphi_1)}/A_l(t)^{(-\varphi)} > \kappa c_E(t)^{\alpha_2(\epsilon-1)}\eta_l/\eta_e$.

Similarly, under Assumption 2 the e-sector's sufficient advancement at time t induces research in the e-sector (l-sector) only for $\epsilon > 1$ ($\epsilon < 1$).

Assumption 3. $A_e(t)^{(-\varphi_1)}/A_l(t)^{(-\varphi)} = \kappa c_E(t)^{\alpha_2(\epsilon-1)}\eta_l/\eta_e$.

Assumption 3 implies that research is directed to both sectors. For the analysis, we use natural baseline scenarios, namely research directed to one sector only for $\epsilon > 1$ and research directed to both sectors in case of $\epsilon < 1$. The intuition follows from Lemma 1. If both sectoral goods are gross substitutes and Assumption 1 holds, research is and will remain directed to the l-sector, as research increases the relative profitability of innovation in this sector through the direct productivity effect that dominates for $\epsilon > 1$. Similarly, when Assumption 2 holds, research is directed to the energy-intensive sector only and further increases the profitability of innovation in the e-sector.

In contrast, when both goods are gross complements and Assumption 1 holds, i.e. the labour-intensive sector is more advanced, research will be directed to the less advanced e-sector as the price effect dominates. Similarly, if Assumption 2 holds, research is directed to the more backward l-sector. Hence, ultimately the equilibrium must be characterised by innovation in both sectors.[8]

3. Energy Intensity Dynamics

After characterising the model equilibrium and the determinants of the direction of technological progress, we analyse the energy intensity of the whole economy. We first show that the evolution of the energy intensity can be disaggregated in two driving forces: a *structural*

[8]This result is formally derived in Appendix C.

effect and an *efficiency effect*. Subsequently, we analyse direction and magnitude of these effects given energy price growth, technical change in the labour-intensive sector, and technical change in the energy-intensive sector. Finally, we combine these results to examine the energy intensity dynamics in heterogeneous economies that differ with respect to their sectoral productivities and the direction of technical change. In order to simplify notation, the time index t is dropped throughout this section.

3.1. Decomposition into Structural Effect and Efficiency Effect

Defining the energy intensity as total energy input relative to total output, E/Y, and using the production function for the final product (2), the energy intensity of the whole economy can be written as:

$$\frac{E}{Y} = \frac{E}{\left(Y_l^{\frac{\epsilon-1}{\epsilon}} + Y_e^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}} = \frac{E}{Y_e}\left(\left(\frac{Y_l}{Y_e}\right)^{\frac{\epsilon-1}{\epsilon}} + 1\right)^{\frac{\epsilon}{1-\epsilon}}. \tag{11}$$

The growth rate of the energy intensity, $\gamma_{\frac{E}{Y}}$, is obtained by taking logarithms and differentiating with respect to time as

$$\underbrace{\gamma_{\frac{E}{Y}}}_{\text{total effect}} \equiv \frac{d\ln\left(\frac{E}{Y}\right)}{dt} = \underbrace{\gamma_{\frac{E}{Y_e}}}_{\text{efficiency effect}} + \underbrace{\left(-\frac{Y_l^{\frac{\epsilon-1}{\epsilon}}}{Y_l^{\frac{\epsilon-1}{\epsilon}} + Y_e^{\frac{\epsilon-1}{\epsilon}}}\right)\gamma_{\frac{Y_l}{Y_e}}}_{\text{structural effect}}, \tag{12}$$

where $\gamma_{\frac{E}{Y_e}}$ denotes the growth rate of the energy intensity in the energy-intensive sector and $\gamma_{\frac{Y_l}{Y_e}}$ is the growth rate of the labour-intensive sector relative to the energy-intensive sector. As shown in expression (12), the development of the energy intensity can be decomposed into an efficiency effect and a structural effect. The efficiency effect refers to changes in the energy intensity in the e-sector. Since only this sector uses energy, any changes in the energy intensity translate directly into the energy intensity of the whole economy. The structural effect captures the relative size of the labour-intensive sector. Since this sector does not use any energy for production, an increase of the share of the labour-intensive sector in total production leads, c.p., to a reduction of the economy wide energy intensity.

Using the previously derived equilibrium values, the strength and direction of the efficiency and the structural effect can be analysed. Using the equilibrium values for energy use and production in the e-sector, (B.21) and (B.23), we can analyse how the energy intensity in the e-sector is affected by changes of the energy costs as well as changes of the productivity levels in both sectors. The equilibrium energy intensity in the energy-intensive sector is:

$$\frac{E}{Y_e} = \frac{\alpha_2 \alpha^{2\alpha} c_E^{\alpha_2 - 1} A_l^{1-\alpha}}{\left(\left(\alpha^{2\alpha} c_E^{\alpha_2}\right)^{1-\epsilon} A_l^{\varphi} + \left(\psi^{\alpha_2} \left(\alpha_1\right)^{2\alpha_1} \left(\alpha_2\right)^{\alpha_2}\right)^{1-\epsilon} A_e^{\varphi_1}\right)^{-\frac{1}{1-\epsilon}}}. \tag{13}$$

Taking the logarithms and differentiating with respect to time yields the following expression for the development of the energy intensity in the energy-intensive sector, i.e. the efficiency effect:

$$\textit{efficiency effect} \equiv \gamma_{\frac{E}{Y_e}} = -\left(1 - \alpha_2 S\right) \gamma_{c_E} + S\left[(1-\alpha)\gamma_{A_l} - (1-\alpha_1)\gamma_{A_e}\right], \tag{14}$$

with $S \equiv \frac{A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} = Y_l^{\frac{\epsilon-1}{\epsilon}} / \left(Y_l^{\frac{\epsilon-1}{\epsilon}} + Y_e^{\frac{\epsilon-1}{\epsilon}}\right) \in (0,1)$, $A \equiv \left(\frac{A_e^{1-\alpha_1}}{A_l^{1-\alpha}}\right)$, $\theta \equiv \left(\frac{\alpha^{2\alpha}}{\psi^{\alpha_2} \alpha_1^{2\alpha_1} \alpha_2^{\alpha_2}}\right)^{1-\epsilon} > 0$, γ_{c_E} denoting the growth rate of the energy price, and γ_{A_l} (γ_{A_e}) denoting the rate of technical change in the labour-intensive (energy-intensive) sector.

In a next step, we derive the structural effect. Using the equilibrium values for sectoral outputs, (B.22) and (B.23), the relative output of the labour-intensive sector is:

$$\frac{Y_l}{Y_e} = \alpha^{2\alpha\epsilon} \alpha_1^{-\frac{2\alpha_1}{1-\alpha}(\epsilon - \epsilon\alpha} \alpha_2^{-\frac{\alpha_2 \epsilon (1+\alpha)}{1-\alpha}} \psi^{\frac{\alpha_1 \alpha_2 \epsilon}{1-\alpha}} A_e^{-\frac{1-\alpha_1}{1-\alpha}(1-\alpha-\varphi)} A_l^{1-\alpha-\varphi} c_E^{\epsilon\alpha_2}. \tag{15}$$

Taking the logarithms, differentiating with respect to time, and multiplying with $(-S)$ yields the structural effect:

$$\textit{structural effect} \equiv -S \cdot \gamma_{\frac{Y_l}{Y_e}} = S \cdot \epsilon \left(-\alpha_2 \gamma_{c_E} - (1-\alpha)\gamma_{A_l} + (1-\alpha_1)\gamma_{A_e}\right). \tag{16}$$

3.2. The Effects of Technical Change and Energy Price Growth

In order to characterize the effect of technical change and energy price growth on energy intensity dynamics, we substitute the expressions (14) and (16) into (12). This yields the growth rate of the economy wide energy intensity as the sum of the efficiency effect (EE) and the structural effect (SE):

$$\begin{aligned} \gamma_{\frac{E}{Y}} = {} & [\underbrace{-\left(1-\alpha_2 S\right)}_{\text{EE}} \underbrace{-S\epsilon\alpha_2}_{\text{SE}}]\gamma_{c_E} + [\underbrace{(1-\alpha)S}_{\text{EE}} \underbrace{-S(1-\alpha)\epsilon}_{\text{SE}}]\gamma_{A_l} \\ & + [\underbrace{-(1-\alpha_1)S}_{\text{EE}} \underbrace{+S\epsilon(1-\alpha_1)}_{\text{SE}}]\gamma_{A_e}. \end{aligned} \tag{17}$$

This expression for the growth rate of energy intensity (total effect) establishes the following proposition that shows how innovation in the *e*-sector, innovation in the *l*-sector, and energy price growth respectively affect the efficiency and the structural effect.

Proposition 1. *i. Innovation in the e-sector,* $\gamma_{A_e} > 0$*, leads to a positive structural effect and a negative efficiency effect, where, in the case of* $\epsilon > 1$ ($\epsilon < 1$)*, the structural (efficiency) effect dominates the efficiency (structural) effect, i.e. it increases (decreases) the growth rate of energy intensity.*

ii. Innovation in the l-sector, $\gamma_{A_l} > 0$*, leads to a negative structural effect and a positive efficiency effect, where, in the case of* $\epsilon > 1$ ($\epsilon < 1$) *the structural (efficiency) effect dominates the efficiency (structural) effect, i.e. it decreases (increases) the growth rate of the energy intensity.*

iii. A positive (negative) growth rate of the energy price, $\gamma_{c_E} > 0$ ($\gamma_{c_E} < 0$)*, leads to a negative (positive) structural effect and a negative (positive) efficiency effect and hence always decreases (increases) the growth rate of the energy intensity.*

Proof. See Appendix D. □

The first part of Proposition 1 (*i*) implies that technical change in the energy-intensive sector, c.p., implies a positive structural effect. The increasing productivity in the energy-intensive sector induces a reallocation of labour towards this sector. Hence, the relative size of the *e*-sector increases over time. This restructuring of the economy towards the energy-intensive sector increases energy intensity (positive structural effect). Furthermore, $\gamma_{A_e} > 0$ implies a negative efficiency effect. Due to the increased productivity of the *e*-sector, the sectoral output grows faster than energy input and hence reduces energy intensity.

According to the first part of Proposition 1 (*ii*), innovation in the labour-intensive sector induces, c.p., an increase in average productivity in the *l*-sector and a reallocation of labour from the *e*- to the *l*-sector. The resulting restructuring of the economy's composition towards the *l*-sector yields a negative structural effect, i.e. a reduction of energy intensity in the economy. The induced decrease in labour input in the *e*-sector causes a substitution of labour by other factors of production, as energy, which, c.p., yields a positive efficiency effect, i.e. an increase of the energy intensity (positive efficiency effect).

In the case of both $\gamma_{A_e} > 0$ and $\gamma_{A_l} > 0$, the structural effect dominates the efficiency effect in the case of substitutes and vice versa for gross complements (second parts of Proposition 1, *i* and *ii*). This result is solely driven by the effect of ϵ on the structural effect, which is reduced, when both sectors are gross complements. Consider gross complements. As can be seen in the relative demand for both sectoral goods (B.1), an increase of output in the *l*-sector induced by $\gamma_{A_l} > 0$ results in a more than proportional increase of the relative price of the energy-intensive good (p_e/p_l) due to the gross complementarity of both sectors. This price reaction dampens the growth of the l-sector and hence the induced structural effect. Similarly, output growth in the e-sector induced by $\gamma_{A_e} > 0$ induces a more than proportional increase in the relative price of the labour-intensive good and also dampens the structural effect. In the case of gross substitutes, the structural effect dominates the efficiency effect. In the case of $\gamma_{A_e} > 0$, e.g., this implies that, in spite of technological improvements in the energy-intensive sector, the increase of the share of this sector's output overcompensates the energy saving effect of technical change and hence leads to an increase of the energy intensity. Similarly, for $\gamma_{A_l} > 0$, the reduction of energy intensity induced by the negative structural effect is stronger than the positive efficiency effect.

Finally, Proposition 1 (*iii*) implies that increasing energy prices, c.p., negatively affect both effects independent of the elasticity of substitution. Positive energy price growth induces a substitution of energy by other factors of production in the energy-intensive sector (negative efficiency effect). The reduction of energy use in the e-sector reduces the marginal productivity of labour in this sector. Hence, labour is reallocated towards the l-sector, which increases the l-sector's relative size (negative structural effect).

3.3. Combined Results

In this subsection, we now turn to the comprehensive analysis of energy intensity developments. We combine the results from Lemmas 1 and 2 and Proposition 1 to examine the joint effect of energy price growth and technical change on the direction and magnitude of the efficiency and the structural effect. The analysis distinguishes between economies that are technologically more advanced in the labour-intensive and those more advanced in the energy-intensive sector. For both cases, we derive how the development of overall energy intensity (total effect) is affected by efficiency and structural effect and different energy price growth rates. We analyse both the case of gross substitutes, $\epsilon > 1$, as well gross complements, $\epsilon < 1$.

Substituting for S in (17) and rearranging yields the growth rate of energy intensity:

$$\gamma_{\frac{E}{Y}} = \left[\frac{\alpha_2(1-\epsilon)\,A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}} - 1\right]\gamma_{c_E} + \left[\frac{\varphi\,A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma_{A_l} + \left[\frac{-\varphi_1\,A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma_{A_e}. \tag{18}$$

The expressions for the efficiency effect (14), the structural effect (16), and the total effect (18) are the basis for the following propositions, which identify the direction and magnitude of these effects for different directions of technical change.

Proposition 2. *With research directed to the l-sector only, i.e. Assumption 1 (Assumption 2) holds for* $\epsilon > 1$ ($\epsilon < 1$)*, and hence* $\gamma_{A_l} = \gamma\eta_l$ *and* $\gamma_{A_e} = 0$*:*

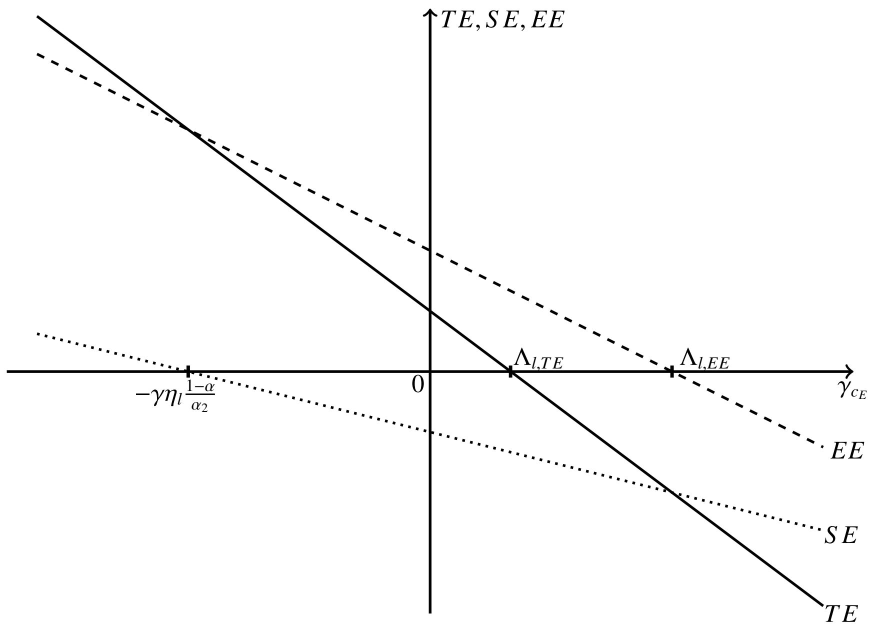

Figure 1: Efficiency, structural, and total effect for $\epsilon > 1$ and research directed to the labour-intensive sector.

i. The total effect is negative, when $\gamma_{c_E} > \frac{\varphi A^{1-\epsilon}}{(\alpha_2(1-\epsilon)-1)\,A^{1-\epsilon}-\theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_l \equiv \Lambda_{l,TE} \overset{(<)}{>} 0 \Leftrightarrow \epsilon \overset{(<)}{>} 1$*, where* $\frac{\partial \Lambda_{l,TE}}{\partial A} < 0$.

ii. The efficiency effect is negative, when $\gamma_{c_E} > \frac{(1-\alpha)A^{1-\epsilon}}{(1-\alpha_2)A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_l \equiv \Lambda_{l,EE} > 0$*, where* $\frac{\partial \Lambda_{l,EE}}{\partial A} \overset{(>)}{<} 0$ *for* $\epsilon \overset{(<)}{>} 1$.

iii. The structural effect is negative, when $\gamma_{c_E} > -\eta_l\gamma\,(1-\alpha)\,/\alpha_2$*, i.e. strong negative growth of the energy price.*

Proof. See Appendix D. □

Assume $\epsilon > 1$ and consider an economy where research is directed entirely to the l-sector, i.e. Assumption 1 holds, and hence Proposition 2 can be applied. In order to illustrate the results, Figure 1 depicts efficiency, structural, and total effect as a function of the energy price growth rate.

The figure shows that energy price growth negatively affects energy intensity development. Furthermore, the evolution of energy intensity is largely driven by the structural effect. As long as the energy price does not decline at a strong rate, the restructuring of the economy away from the energy-intensive sector has a decreasing effect on the overall energy intensity. The efficiency effect becomes negative for all energy price growth rates above the threshold $\Lambda_{l,EE}$. When the energy price grows at a lower rate or even decreases, producers in the energy-intensive sector do not have incentives to reduce energy use. In addition, technical change in the l-sector induces a reallocation of labour towards this sector and hence fosters a substitution of labour by other factors of production, as energy, in the e-sector. The threshold $\Lambda_{l,EE}$ is negatively affected by A. As research is directed to the l-sector only, A declines and hence the threshold $\Lambda_{l,EE}$ increases. The intuition is as follows. A higher productivity in the l-sector induces a reallocation of labour towards this sector. The reduction of labour in the e-sector fosters a substitution away from labour towards other production factors, as energy. Hence, the higher the productivity advantage of the l-sector, the higher the energy price growth rate has to be in order to induce a negative efficiency effect. The total effect is negative, when energy price grows at a rate larger than $\Lambda_{l,TE}$.

Proposition 3. *With research directed to the e-sector only, i.e. Assumption 2 (Assumption 1) holds for* $\epsilon > 1$ *(*$\epsilon < 1$*), and hence* $\gamma_{A_l} = 0$ *and* $\gamma_{A_e} = \gamma\eta_e$*:*

i. The total effect is negative, when $\gamma_{c_E} > \frac{\varphi_1 A^{1-\epsilon}}{(\alpha_2(1-\epsilon)-1)A^{1-\epsilon} - \theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_e \equiv \Lambda_{e,TE} \overset{(<)}{>} 0 \Leftrightarrow \epsilon \overset{(<)}{>} 1$*, where* $\frac{\partial \Lambda_{e,TE}}{\partial A} < 0$.

ii. The efficiency effect is negative, when $\gamma_{c_E} > -\frac{(1-\alpha_1)A^{1-\epsilon}}{(1-\alpha_2)A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_e \equiv \Lambda_{e,EE} < 0$*, where* $\frac{\partial \Lambda_{e,EE}}{\partial A} \overset{(<)}{>} 0 \Leftrightarrow \epsilon \overset{(<)}{>} 1$.

iii. The structural effect is negative, when $\gamma_{c_E} > \eta_e\gamma(1-\alpha_1)/\alpha_2$*, i.e. strong growth of the energy price.*

Proof. See Appendix D. □

In contrast, consider an economy that is more advanced in the energy-intensive sector, i.e. Assumption 2 holds, and research is directed to e-sector. In this case, the results of Proposition 3 can be applied, which are illustrated in Figure 2.

Similar to the previous case, the figure clearly shows that the development of energy intensity and both partial effects are negatively affected by the energy price growth rate. In contrast to the case of technical change directed to the l-sector, the structural effect is positive as long as there is no strong growth of the energy price. Technical change in the energy-intensive sector induces a structural change of the economy towards this sector. Hence, the structural effect is mostly positive in this case. In contrast, the efficiency effect is negative for all energy price

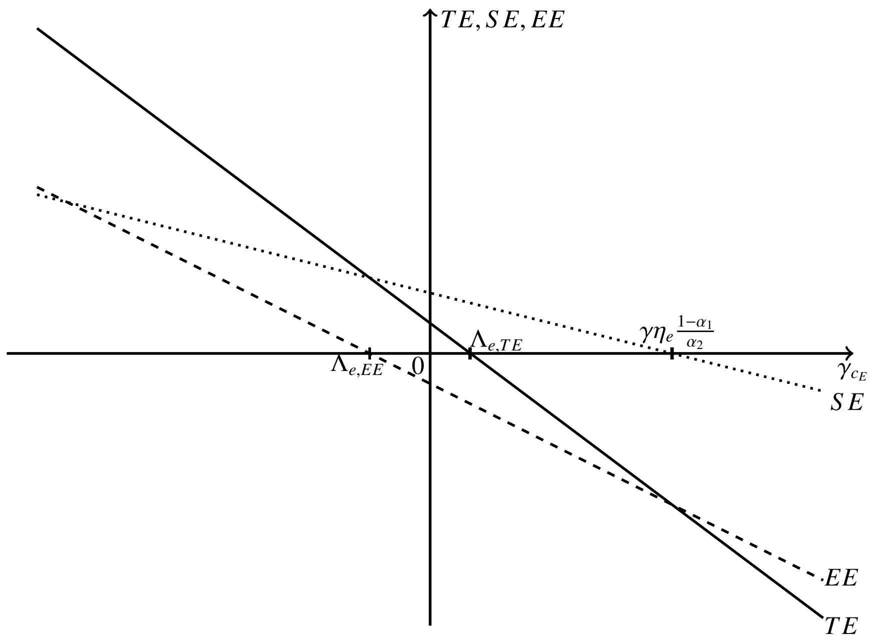

Figure 2: Efficiency, structural, and total effect for $\epsilon > 1$ and research directed to the energy-intensive sector.

growth rates above the negative threshold $\Lambda_{e,EE}$ and largely drives the energy intensity development. The threshold itself is positively affected by A, and hence increases, when research is directed to the e-sector only. The higher the productivity in the e-sector, the more labour is reallocated towards this sector and the more costly it becomes to attract additional labour from the l-sector. This increases the incentive for producers in the e-sector to substitute away from labour towards other factors of production, as energy. Hence, the higher A, the larger $\Lambda_{e,EE}$ has to be in order to induce a negative efficiency effect. For energy price growth rates above $\Lambda_{e,TE}$, the negative efficiency effect dominates the positive structural effect and hence the total effect is negative.

Comparing the efficiency effects in Figures 1 and 2 illustrates the role of technical change in the e-sector. In the case depicted in Figure 1, there is no technical change in the energy-intensive sector. Hence, the negative efficiency effect is solely caused by the substitution of energy by other factors of production, which is only induced by energy price growth above $\Lambda_{l,EE}$. In Figure 2 we can see that even for small negative energy price growth rates the efficiency effect is negative, which is due to the additional effect of technical change in the energy-intensive sector in this case.

For $\epsilon > 1$, it is important to bear in mind that, as outlined in Lemma 2, a sufficiently strong (negative) energy price growth can ultimately change the direction of research. In the case of an economy, where research is initially directed to the e-sector (Proposition 3), strong growth of the energy price will ultimately induce a redirection of innovation towards the labour-intensive sector. This effect can be seen Figure 2, where the structural effect becomes negative for $\gamma_{c_E} > \eta_e \gamma (1 - \alpha_1) / \alpha_2$. The intuition is as follows. The rapidly growing costs of energy cannot be compensated by innovation. Energy input declines over time and hence the output in the energy-intensive sector shrinks. This means that strong energy price growth fosters a restructuring of the economy towards the l-sector even when innovation is still directed to the e-sector. As the decline in relative output is stronger than the increase of its relative price, the profitability of innovation in this sector decreases. This process continues until the relative profitability of research in the e-sector falls below unity and research switches to the l-sector, i.e. Assumption 1 applies. The timing of this switch of research depends, next to the actual magnitude of energy price growth, on the relative productivity of the e-sector.[9]

Proposition 4. *Consider $\epsilon < 1$.*

i. With moderate growth of the energy price growth, i.e. $-\eta_l \gamma (1 - \alpha) / \alpha_2 \leq \gamma_{c_E} \leq \eta_e \gamma (1 - \alpha_1) / \alpha_2$, and research directed to the both sectors, i.e. Assumption 3 holds, and hence $\gamma_{A_l} = s_l \gamma \eta_l = \frac{\alpha_2(\epsilon-1)\frac{\gamma_{c_E}}{\gamma} + \eta_e \varphi_1}{\eta_e \varphi_1 + \eta_l \varphi} \gamma \eta_l$ and $\gamma_{A_e} = s_e \gamma \eta_e = \frac{\alpha_2(1-\epsilon)\frac{\gamma_{c_E}}{\gamma} + \eta_l \varphi}{\eta_e \varphi_1 + \eta_l \varphi} \gamma \eta_e$, the efficiency effect equals $-\gamma_{c_E}$, the structural effect equals zero. Hence, the total effect equals $-\gamma_{c_E}$.

ii. With strong growth of the energy price growth, i.e. $\gamma_{c_E} > \eta_e \gamma (1 - \alpha_1) / \alpha_2$, and research directed to the the e-sector only, i.e. Assumption 1 holds, and hence $\gamma_{A_l} = 0$ and $\gamma_{A_e} = \gamma \eta_e$, the efficiency effect, the structural effect, and the total effect are negative.

iii. With strong negative growth of the energy price growth, i.e. $\gamma_{c_E} < -\eta_l \gamma (1 - \alpha) / \alpha_2$, and research directed to the the l-sector only, i.e. Assumption 2 holds, and hence $\gamma_{A_l} = 0$ and $\gamma_{A_e} = \gamma \eta_e$, the efficiency effect, the structural effect, and the total effect are positive.

Proof: See Appendix D.

The results of Proposition 4 are illustrated in Figure 3. When research is directed to both sectors, which is the relevant case for moderate energy price growth rates, the relative sector size does not change and hence the structural effect is equal to zero. The evolution of energy intensity is solely driven by the efficiency effect. The higher the energy price growth, the larger the share of scientists directing their research towards the e-, which is the typical result of DTC models.[10] This reallocation compensates the increasing growth rate of the energy price such

[9]The reverse effect applies for an economy, where research is initially directed to the l-sector. In this case, strong negative growth of the energy price will ultimately redirect innovation to the e-sector.

[10]This can be seen in the expressions for the equilibrium allocation of researchers to the l- and the e-sector, s_l and s_e, in Proposition 4. As can be seen for $\epsilon < 1$, s_e increases, when γ_{c_E} decreases, i.e. the higher energy price growth, the more researchers direct their effort towards the energy-intensive sector.

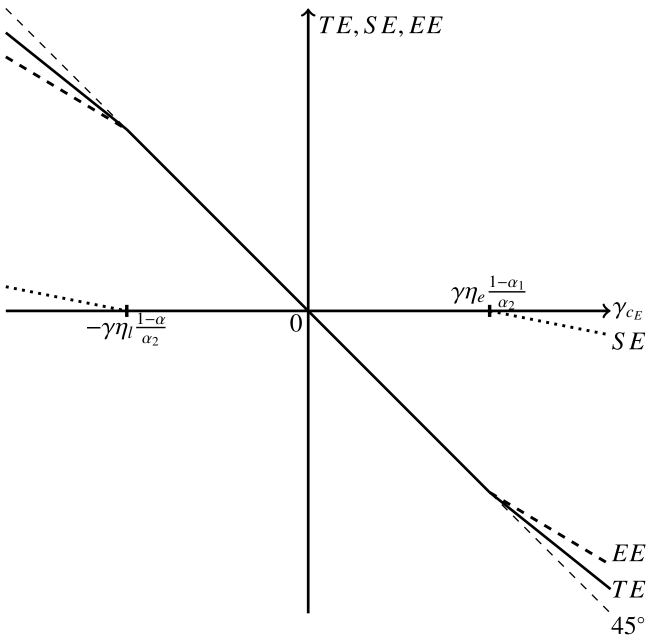

Figure 3: Efficiency, structural, and total effect for $\epsilon < 1$.

that the relative sector size remains constant (structural effect is zero). Increasing costs of energy induce substitution of energy by other production factors in the energy-intensive sector which leads to a negative efficiency effect. Given the constant relative sector size, this directly translates to a energy intensity reduction in the whole economy. The opposite effects can be observed for moderate negative growth rates of the energy price.

For strong (negative) growth rates of the energy price, research is directed to one sector only. In case of strong energy price growth, i.e. $\gamma_{c_E} > (1 - \alpha_1)\, \eta_e \gamma / \alpha_2$, all research is directed to the e-sector, i.e. Assumption 1 holds. However, technical change in the energy-intensive sector cannot overcompensate the rapidly growing energy costs. This means that strong energy price growth fosters a restructuring of the economy towards the l-sector even when innovation is still directed to the e-sector. Hence, as we can see in Figure 3, the structural effect becomes negative.[11]

Overall, the model results crucially depend on whether ϵ is larger or smaller than unity, which is a typical attribute of DTC models. Our theoretical results better fit empirical observations for $\epsilon > 1$. For $\epsilon < 1$, the structural effect is equal to zero, unless there is a strong positive or negative growth rate of the energy price. However, decomposition analyses show an important role of sectoral adjustments as a driver of energy intensity reductions as they

[11]The opposite effects can be observed for strong negative energy price growth. Research is ultimately directed to the l-sector only inducing a positive structural effect.

attribute for 25% of energy intensity reduction in OECD countries (Mulder and de Groot, 2012). For gross substitutes, the model predicts an efficiency and a structural effect different from zero for almost any energy price growth rate. The latter results are in line with empirical decomposition studies, that typically find both effects. It is, however, difficult to provide empirical evidence on this elasticity. While there are numerous studies estimating elasticities of substitution between production factors, there are almost no estimates of elasticities of substitution between sectors. Exceptions are Oberfield and Raval (2014), who estimate cross industry elasticities of demand based on US data and overall find values ranging between 0.75 and 2.2, and Edmond et al. (2015), who determine an elasticity of substitution across sectors of 1.24. To our knowledge, however, there are no estimates of the elasticity of substitution between energy-intensive and labour-intensive sectors and an estimation of such an elasticity is out of the scope of this paper. Overall, we consider $\epsilon > 1$ to be the more plausible assumption, which we will use for the subsequent calibration.

4. Cross-Country Differences in Energy Intensity Dynamics

In this section, we present simulations of energy intensity developments and their drivers across countries. The purpose of this exercise is not to provide comprehensive quantitative predictions of energy intensity developments. Our objectives are twofold. First, we illustrate the main results of our theoretical model, i.e. how differences in sectoral productivities and different energy prices between countries affect energy intensity dynamics in the model. Second, we cross-check our results with empirical decomposition studies.

4.1. Calibration

We calibrate the model based on the World Input-Output Database (WIOD, 2013).[12] Our calibration mainly draws from the Environmetal Accounts (EA) and the Socio Economic Accounts (SEA) of the WIOD both covering 34 sectors in 40 countries from 1995 - 2007/2009. As we explicitly model energy as an input factor in the energy-intensive sector, we drop two energy producing sectors from the WIOD in the calibration, namely *Coke, Refined Petroleum and Nuclear Fuel* (WIOD Code 23) and *Electricity, Gas and Water Supply* (WIOD Code E). An advantage of this data is that it contains consistent information relevant for the calibration. Furthermore, it benefits the cross-checking of our results with the decomposition study of Voigt et al. (2014), which is also based on the WIOD. For the energy price, we use the Indices of Real Energy Prices for Industry from the IEA as they are based on energy prices paid by

[12] We use the 2013 release of the data, which is available at http://www.wiod.org. For detailed information on data sources, construction, and structure of the database see Dietzenbacher et al. (2013), Genty et al. (2012), and Timmer et al. (2015).

firms (IEA, 1999, 2007, 2008, 2017).[13] Combining both sources yields a sample of 32 sectors in 26 OECD countries between 1995 and 2007 (see Table 1 in Appendix A for an overview). We calibrate the model based on 1995 data and simulate the development of energy intensities and its drivers until 2007.

As we use a two-sector model, all sectors covered in the WIOD have to be aggregated into two sectors, i.e. an energy-intensive and a labour-intensive sector. We use data on sectoral energy use (EU) in physical units (TJ) from the EA and sectoral gross output (GO) in million USD from the SEA for all 26 countries in order to calculate the aggregate energy intensity for each sector. We calculate the average energy intensity and define all sectors with energy intensities above the average as energy-intensive, while all sectors with energy intensities below the average are aggregated into the labour-intensive sector.

We take $\eta_e = \eta_l = 0.02$ and $\gamma = 1$, which is consistent with a long-run growth rate of 2% (Acemoglu et al., 2012; Acemoglu et al., 2015). We follow the standard convention to set the labour share of income to $(1 - \alpha) = 2/3$. Hence, a share of $\alpha = 1/3$ is spent on machines, which could be interpreted as capital, in the l-sector and in the e-sector on both machines and energy. For the latter sector, we need to also calibrate α_2, which is the energy share of output in the energy-intensive sector. For each country, we derive the energy costs at purchasers' prices in the energy-intensive sector from the World Input-Output Tables of the WIOD.[14] Using the data on sectoral GO, we then calculate the energy cost share in the e-sector for each country, which gives us proxies of α_2 for each country.[15] We further set $\epsilon = 2$.

Finally, we need to determine the initial sectoral productivities $A_e(t = 0)$ and $A_l(t = 0)$ for all countries in 1995. While these are difficult to observe, the SEA contain information on sectoral employment (total hours worked by persons engaged), which allows us to compute employment in the l-sector relative to the e-sector. Using the relative employment condition (B.16), we can then set the sectoral productivities to match the observed employment. Hence, also the direction of research is determined for each country.

4.2. Results

Given the Parameter choices outlined above, we simulate the efficiency, the structural, and the total effect for all countries in our sample. Figure 4 shows the correlation between the average

[13] Similar to our approach, Ley et al. (2016) base their analysis of the effect of energy prices on green innovation on end-use energy prices for the manufacturing sector from IEA.

[14] Similar to Kaltenegger et al. (2017), we calculate energy costs as a sum of four cost components: (i) coal, lignite, and peat (CPA10), (ii) crude petroleum and natural gas; services incidental to oil and gas extraction excluding surveying (CPA11), (iii) coke, refined petroleum products and nuclear fuels (CPA23), and (iv) electrical energy, gas, steam and hot water (CPA40).

[15] We calibrate α_2 country-specific, as we see quite substantial cross-country differences in the energy cost shares ranging from below 4% to more than 15%.

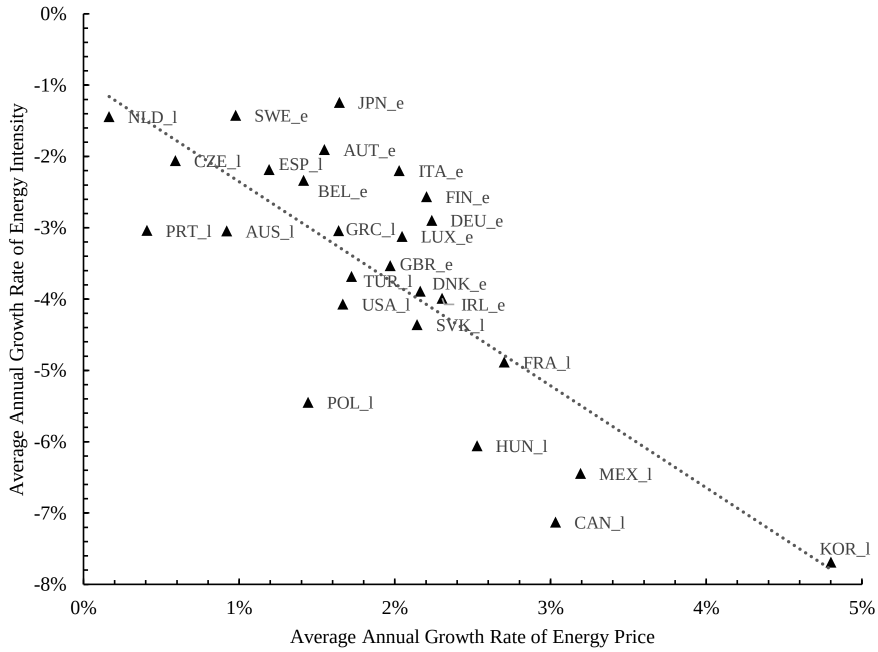

Figure 4: Correlations between average annual growth rates of energy prices and energy intensities. The subscript e (l) denotes the direction of technical change towards the e-sector (l-sector).

annual growth rate of energy prices and energy intensity for all 26 countries. The figure illustrates some core results of the model. The higher the growth rate of the energy price, the stronger is the reduction of energy intensity. The overall reduction of energy intensity seems, on average, larger in countries, where technical change is directed towards the labour-intensive sector.

The results for the efficiency effect and the structural effect are depicted in Figure 5. The figure shows the average annual growth rates for both effects based on our simulation. To cross-check our results, we also calculated the respective growth rates based on the decomposition analysis by Voigt et al. (2014), who cover all 26 countries that we analyse in their decomposition study. Furthermore, they use WIOD data for their analysis, which is the basis for our calibration.

In 11 out of the 26 countries, research is directed towards the energy-intensive sector, i.e. Proposition 3 holds, and hence energy intensity dynamics should be dominated by the efficiency effect. This can be seen, e.g., for Germany. While the structural effect is positive, the efficiency effect is negative. As the growth rate of the energy price is above the threshold $\Lambda_{e,TE}$, the total effect is negative. The restructuring towards the energy-intensive sector induced by innovation in this sector positively affects energy intensity, but is overcompensated by the energy savings within the e-sector. In contrast, the USA are an example for an economy that is

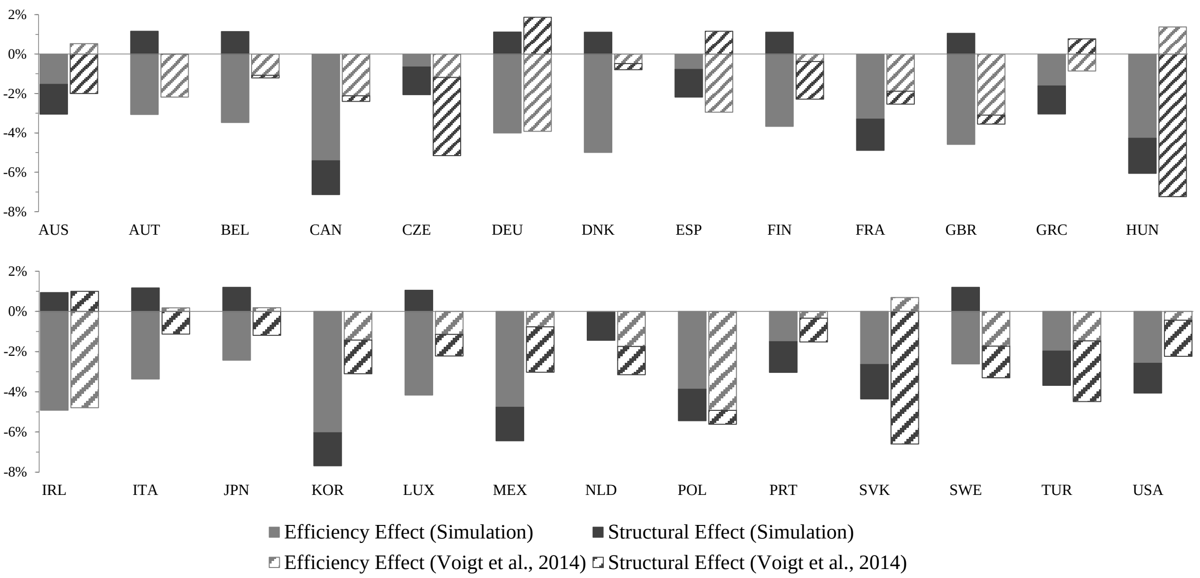

Figure 5: Efficiency effect and structural effect - Simulation Results and Results of Voigt et al. (2014)

relatively more advanced in the labour-intensive sector, i.e. Proposition 2 holds. As research is directed to the *l*-sector, we see a negative structural effect in the USA. The efficiency effect is negative as well, which implies an energy price growth rate above the growth rate of the energy price is above the threshold $\Lambda_{l,EE}$.

In both cases, our model's predictions are in line with the decomposition of Voigt et al. (2014). In some cases, our simulation results contradict their results. For Sweden, e.g., our model predicts research to be directed towards the energy-intensive sector resulting in a positive structural and a negative efficiency effect, whereas Voigt et al. (2014) finds a negative structural effect. The opposite discrepancy can be observed for Spain. In the latter case, however, our results are in line with Mulder and de Groot (2012), who find a negative structural effect for Spain in a similar period and hence are in line with our predictions.[16]

Overall, the simulation results of our stylised model seem to be largely consistent with the decomposition studies. To examine the sensitivity of our results, we used different methods to aggregate all available sectors into the energy-intensive and the labour-intensive sector. In a first alternative calibration, we only considered sectors with energy intensities of at least (not more than) 10% above (below) the average as energy-intensive (labour-intensive). We repeated this for 25% and 50%. Hence, going from 10% to 50%, we excluded more sectors that are close to the average of the energy intensity, i.e. we focused more on sectors with very

[16] Table 2 in Appendix A depicts an overview of our simulation results, the results of Voigt et al. (2014) and additionally those of Mulder and de Groot (2012), who computed average annual growth rate for all three effects between 1995 and 2005.

high and those with very low energy intensities. We also used sectoral energy costs per gross output, i.e. sectoral energy cost shares, as an alternative measure to split the sectors into the two groups.[17] Similar to energy intensities, we also stepwise excluded sectors close to the average cost share and repeated the calibration exercise. Overall, the simulation results stayed qualitatively stable compared to our baseline scenario, which we consider to be the most suited for our model.

5. Discussion

Our model provides insights on the impacts of energy prices and technical change on the development of energy intensity and, in particular, the relative importance of structural adjustments between sectors and energy efficiency improvements within sectors. Furthermore, our model predicts a negative effect of energy price growth on economy-wide energy intensity, which is in line with empirical evidence (Löschel et al., 2015; Metcalf, 2008; Moshiri and Duah, 2016; Popp, 2002). Our simulations illustrate how these two effects predicted by our model vary between countries.

We show that energy intensity reductions are driven by the efficiency effect, when research is directed to the energy-intensive sector, which can be seen in the simulation results for, e.g., Austria or Germany. This efficiency effect is driven by technical change in the *e*-sector as well as factor substitution induced by energy price growth, which is in line with empirical findings. Fisher-Vanden et al. (2016) empirically investigate energy intensive industries and find that higher energy prices and R&D stocks negatively affect energy intensity in these industries. Steinbuks and Neuhoff (2014) analyse various industries and show that the effect of energy price is higher for energy intensive industries and that labour is a substitute for energy. Wang (2013) conducts a decomposition of the efficiency effect in underlying driving forces and shows that technical progress is the the main contributor to energy intensity reductions in Europe. According to Popp (2001), two thirds of the energy savings in energy-intensive industries are due to factor substitution, while one third is due to innovation.

When research is directed to the labour-intensive sector, the model predicts that structural adjustments are a main driver of energy intensity developments. Examples for this case are France or the USA in our simulation exercise. According to our model, the efficiency effect is negative as well if the energy price growth rate is above the positive threshold $\Lambda_{l,EE}$, which is the case for all the countries in our sample. However, the results of the long-run decomposition study by Sue Wing (2008), covering the second half of the 20th century, provide some further evidence on the relationship between $\Lambda_{l,EE}$ and efficiency effect in this case. Sue Wing (2008)

[17]See footnote 14 for the approach to calculate sectoral energy costs.

decomposes energy intensity in the USA and shows that, in the period between 1958 and the energy price shock 1974-1986, energy price was decreasing and the efficiency effect in the USA was positive. According to our model, the efficiency effect is positive if the energy price growth rate is below the positive threshold $\Lambda_{l,EE}$, which is in line with the empirical evidence. For energy price growth rates below the threshold, there are no incentives to substitute away from energy and hence the efficiency effect is positive. In the period we analyse, the average energy price growth rate is above $\Lambda_{l,EE}$ and hence induces substitution away from energy and resulting in a negative efficiency effect.

We further show that strong (negative) energy price growth may redirect technical change. In our sample, however, we did not observe strong positive or negative growth rates of the energy price.[18] The scenario of strong energy price growth could be applied to the periods of the energy crises and their aftermath (1974-1986) that were characterised by dramatic increases in energy costs (Alpanda and Peralta-Alva, 2010; Linn, 2008; Sue Wing, 2008). In case of gross substitutes, our model predicts stronger energy intensity reductions for strong energy price growth. This finding is in line with, e.g. Sun (1998), who analyses the period 1973-1990 and shows that the reduction in energy intensity was particularly strong in the periods 1973-1980 (14.25%) and 1980-1985 (12.52%). Although this period of strong energy price growth was temporary, it could have redirected technological progress from the energy-intensive to the labour-intensive sector, as outlined in Section 3.

Our model is highly stylized compared to the complex reality. We used some simplifications in order to identify the effects of energy price and directed technical change on energy intensity dynamics as clearly as possible. Hence, there is room for extensions of our approach. One simplifying assumption we used was an exogenous energy price and did not explicitly model energy generation. We think that this assumption is not too critical, as we do not attempt to do an analysis or predictions for the (very) long run. Furthermore, we interpret the energy price as the final energy price faced by producers including all taxes, which are exogenous from the producers' perspective. According to Sato et al. (2015), the cross-country variation in final industrial energy prices is largely explained by variations in the tax component (e.g., around 60% for electricity and 50% - 80% for oil).[19] However, in reality the energy price is not independent of demand. An extension of the model could be to introduce an endogenous energy price by, e.g., introducing resource extraction or an energy production sector. Although we do not model an endogenous energy price, we are able to assess how such an extension would affect our results. In our model, a price growth induces, e.g., a more efficient use of

[18] The period since the late 1980s, particularly since the late 1990s, has been mainly characterised by moderately growing energy prices (Lee and Lee, 2009; Ley et al., 2016; Narayan and Narayan, 2007; Regnier, 2007).

[19] Due to this attribute, end-use energy price indices are used as proxies for environmental policy stringency in empirical studies, as Sato and Dechezleprêtre (2015) and Aldy and Pizer (2015).

energy, which does not have any effects on the energy price. With endogenous energy prices, this price-induced reduction of energy use could in turn dampen the energy price increase. Hence, the introduction of an endogenous energy price in this model would probably reduce the magnitude of the efficiency and structural effects we predict. In order to explicitly analyse the effect of energy taxes/subsidies, the end-use energy price could be split up in a wholesale price and tax/subsidy component ($c_E = c_W + \tau$). Although we assumed the price to implicitly include effects of regulatory instruments, as taxes, we are able to gain some insights on energy-saving policies. A tax on energy, e.g., increases the end-user price of energy and hence negatively affects energy intensity. However, such a policy would mainly work through energy intensity reductions within the energy-intensive sector, which is in line with the findings of Mulder (2015). A redirection of research to the labour-intensive sector would require very high price increases.

For our analysis, we needed a model with at least two sectors that differ in their energy intensity. We followed the majority of the DTC literature by introducing one final good that is assembled from two sectoral goods. This choice, however, does not drive any of the results. The production function for the aggregate output Y could also be interpreted as the households' preferences for sectoral output (Pittel and Bretschger, 2010). To introduce a difference in energy-intensity across sectors, we chose to assume that the productivity of energy in the labour-intensive sector is zero, which reduces the production function of $Y_l(t)$ and hence simplifies the analysis. Such a sectoral structure is commonly used for analyses in two-sector DTC models, where one sector is more energy-, resource-, or emission-intensive than the other (Acemoglu et al., 2012; Daubanes et al., 2013; Di Maria and Valente, 2008; Di Maria and van der Werf, 2008; Pittel and Bretschger, 2010). This simplifying assumption – energy input is only included in one sector – allows for a clear identification of the efficiency and the structural effect and their driving forces. Of course, one might also think of an alternative and more realistic modelling of this sector structure. A possibility could be to introduce energy input in both sectors, but introduce differences between the sectoral production functions to model the difference in energy intensity, e.g., by assuming energy and other factors to be complements in one sector and substitutes in the other. Such extensions, however, might affect the model's tractability and make it more complex, or even impossible, to analytically decompose energy intensity changes into efficiency and structural effect.

6. Conclusion

In this paper, we used a DTC model with an energy-intensive and a labour-intensive sector to analyse the adverse developments of energy intensities across countries. We decomposed energy intensity into a structural effect and an efficiency effect in order to investigate their dynamics due to the direction of research and energy price growth.

Our main contribution to the literature is a first attempt to theoretically analyse the determinants of heterogeneous energy intensity trends based on a dynamic model with endogenous technical change. So far, studies analysing the trends in energy intensities and the interaction of the driving forces, as the structural and efficiency effect, have been empirical. With increasing availability of data and sophisticated methodologies, these studies, particularly those using decomposition methods, have shown extensive and fruitful insights into energy intensity trends that substantially differ across countries. We offer an explanation why structural adjustments drive energy intensity reductions in certain countries whereas they are dominated by within-sector efficiency improvements in others.

We have analysed how energy price growth and the relative productivity of labour- and energy-intensive sectors affect the direction of research and hence the direction and magnitude of the aforementioned two effects. For the case of gross substitutes, we have shown that in economies that are relatively more advanced in the labour-intensive sector, research is directed to this sector and the energy intensity developments are mainly driven by the structural effect. In economies with a relatively more productive energy-intensive sector, the efficiency effect dominates the evolution of energy intensity. When both sectoral goods are gross complements and research is directed to both sectors, energy intensity dynamics are solely driven by the efficiency effect as the relative sector size remains constant. Energy price growth generally negatively affects energy intensity developments and strong positive (negative) growth rates of the energy price can ultimately redirect technical change. Finally, we have calibrated the model to empirical data to illustrate how differences in energy price growth and sectoral productivities affect energy intensity trends across 26 OECD countries. In spite of our very stylised model, the results are largely consistent with empirical studies.

An area of future work might be an empirical investigation of the elasticity of substitution between sectors with high and low energy intensities. As our approach is a first step to theoretically analyse underlying drivers of energy intensity dynamics, extensions or alternative theoretical modelling strategies seem a fruitful direction of further research. In addition to the proposed extensions discussed above, it would be valuable to develop a multi-country model that could be used to analyse between-country structural adjustments caused by international trade, as the data indicates structural adjustments in production between countries. Overall, theoretical research appears to have a potential for important additional insights, as the empirical literature has taught us a great deal about energy intensity developments and its decomposition, whereas the underlying determinants are still largely unexplored.

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A. Appendix

Table 1: Sectoral Energy Intensities

Sector	Energy Intensity*
Real Estate Activities (sec70)	0.49
Financial Intermediation (secJ)	0.55
Transport Equipment (sec34t35)	0.79
Electrical and Optical Equipment (sec30t33)	0.81
Renting of M&Eq and Other Business Activities (sec71t74)	0.99
Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles (sec51)	1.03
Machinery, Nec (sec29)	1.10
Leather, Leather and Footwear (sec19)	1.26
Post and Telecommunications (sec64)	1.33
Manufacturing, Nec; Recycling (sec36t37)	1.37
Health and Social Work (secN)	1.45
Education (secM)	1.45
Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel (sec50)	1.57
Construction (secF)	1.70
Rubber and Plastics (sec25)	1.76
Food, Beverages and Tobacco (sec15t16)	1.84
Other Community, Social and Personal Services (secO)	1.95
Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods (sec52)	2.05
Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies (sec63)	2.12
Hotels and Restaurants (secH)	2.21
Textiles and Textile Products (sec17t18)	2.31
Public Admin and Defence; Compulsory Social Security (secL)	3.15
Agriculture, Hunting, Forestry and Fishing (secAtB)	4.43
Wood and Products of Wood and Cork (sec20)	4.69
Pulp, Paper, Paper , Printing and Publishing (sec21t22)	5.18
Inland Transport (sec60)	6.52
Basic Metals and Fabricated Metal (sec27t28)	7.15
Other Non-Metallic Mineral (sec26)	9.10
Mining and Quarrying (secC)	12.28
Chemicals and Chemical Products (sec24)	15.11
Water Transport (sec61)	22.66
Air Transport (sec62)	24.26

*Energy intensity = energy use / gross output, measured in gross energy use in TJ per millions of US.

Table 2: Efficiency, structural, and total effect across countries (average annual growth rates)

	Simulation			Voigt et al. (2014)			Mulder and de Groot (2012)		
Country	EE	SE	TE	EE	SE	TE	EE	SE	TE
AUT	-3.06	1.16	-1.90	-2.18	0.00	-2.18	-0.20	0.40	0.30
BEL	-3.47	1.14	-2.33	-1.08	-0.13	-1.21	-1.10	-0.50	-1.60
CZE	-0.66	-1.40	-2.06	-1.19	-3.97	-5.15	-1.40	0.40	-1.10
DEU	-4.01	1.11	-2.89	-3.92	1.87	-2.06	-2.10	-0.20	-2.40
DNK	-5.00	1.11	-3.89	-0.49	-0.31	-0.80	-1.90	-1.20	-3.20
ESP	-0.78	-1.40	-2.19	-2.95	1.16	-1.80	3.80	-1.10	2.70
FIN	-3.67	1.11	-2.56	-0.38	-1.91	-2.29	-2.60	-1.50	-4.10
FRA	-3.30	-1.58	-4.88	-1.89	-0.66	-2.55	-1.50	-0.50	-2.00
GBR	-4.58	1.05	-3.53	-3.10	-0.46	-3.56	-0.30	-2.10	-2.40
HUN	-4.28	-1.78	-6.06	1.37	-7.24	-5.86	-5.50	-1.10	-6.60
ITA	-3.37	1.17	-2.20	0.17	-1.13	-0.96	-4.80	-0.70	-5.50
JPN	-2.44	1.19	-1.24	0.18	-1.20	-1.02	0.30	-1.30	-1.00
KOR	-6.04	-1.65	-7.69	-1.44	-1.67	-3.11	2.60	-0.40	2.20
NLD	-0.05	-1.39	-1.44	-1.75	-1.41	-3.15	-1.30	-0.30	-1.70
POL	-3.88	-1.57	-5.45	-4.93	-0.69	-5.62	-1.30	0.50	-0.90
SVK	-2.65	-1.71	-4.36	0.69	-6.61	-5.91	-7.20	1.30	-5.80
SWE	-2.62	1.19	-1.42	-1.73	-1.57	-3.31	-1.40	-2.60	-4.00
USA	-2.58	-1.49	-4.07	-0.44	-1.80	-2.24	-3.40	-0.70	-4.10
AUS	-1.54	-1.51	-3.05	0.52	-2.00	-1.48			
CAN	-5.42	-1.71	-7.13	-2.12	-0.29	-2.41			
GRC	-1.62	-1.42	-3.04	-0.86	0.78	-0.09			
IRL	-4.92	0.93	-3.99	-4.80	1.00	-3.80			
LUX	-4.17	1.05	-3.12	-1.15	-1.07	-2.22			
MEX	-4.78	-1.66	-6.44	-0.78	-2.26	-3.03			
PRT	-1.51	-1.53	-3.04	-0.34	-1.18	-1.52			
TUR	-1.98	-1.71	-3.68	-1.47	-3.02	-4.49			

B. Appendix: Solving for the Equilibrium

In order to simplify notation, we drop the time index in Appendix B. Due to perfect competition on market for the final product, the profit-maximising behaviour of the final good producer results in the following relative demand for both sectoral goods:

$$\frac{p_l}{p_e} = \left(\frac{Y_l}{Y_e}\right)^{-\frac{1}{\epsilon}}. \tag{B.1}$$

This price ratio implies that the relative price is inversely related to the relative supply of both sectors. Defining the final good as numeraire, the price index can be written as

$$\left(p_l^{1-\epsilon} + p_e^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} = 1. \tag{B.2}$$

Sectoral producers maximise their profits by choosing the quantities of the respective sector specific machines and labour,

$$\max_{x_{li},L_l} \left\{ \Pi_{Y_l} = p_l L_l^{1-\alpha} \int_0^1 A_{li}^{1-\alpha} x_{li}^{\alpha} di - wL_l - \int_0^1 p_{li} x_{li} di \right\}, \tag{B.3}$$

as well as, in the case of the e-sector, the amount of energy,

$$\max_{x_{ei},L_e,E} \left\{ \Pi_{Y_e} = p_e E^{\alpha_2} L_e^{1-\alpha} \int_0^1 A_{ei}^{1-\alpha_1} x_{ei}^{\alpha_1} di - wL_e - \int_0^1 p_{ei} x_{ei} di - c_E E \right\}. \tag{B.4}$$

Profit-maximisation yields the sectoral demands for machine i in the labour-intensive sector,

$$x_{li} = \left(\frac{\alpha p_l}{p_{li}}\right)^{\frac{1}{1-\alpha}} L_l A_{li}, \tag{B.5}$$

and in the energy-intensive sector,

$$x_{ei} = \left(\frac{\alpha_1 p_e E^{\alpha_2} L_e^{1-\alpha}}{p_{ei}}\right)^{\frac{1}{1-\alpha_1}} A_{ei}. \tag{B.6}$$

The demands for machines increase in the price of the respective sector's output (p_j), employed labour in the sector (L_j), and the quality of the individual technology (A_{ji}).

Machines are produced under monopolistic competition. The producer of each variety maximises her profit ($\pi_{ji} = \left(p_{ji} - \psi\right) x_{ji}$) given the demand for her variety. The optimisations yield the price setting rules for monopolists in both sectors, that are $p_{li} = \psi/\alpha$ for machine producers in the l-sector and $p_{ei} = \psi/\alpha_1$ for machine producers in the e-sector. Using these prices and the demands for machines in both sectors, (B.5) and (B.6), the equilibrium profits of machine producers in the labour-intensive sector are

$$\pi_{li} = (1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}}\left(\frac{1}{\psi}\right)^{\frac{\alpha}{1-\alpha}} p_l^{\frac{1}{1-\alpha}} L_l A_{li}, \tag{B.7}$$

whereas the profits in the energy-intensive sector are

$$\pi_{ei} = (1-\alpha_1)\,\alpha_1^{\frac{1+\alpha_1}{1-\alpha_1}}\left(\frac{1}{\psi^{\alpha_1}}\right)^{\frac{1}{1-\alpha_1}} p_e^{\frac{1}{1-\alpha_1}} E^{\frac{\alpha_2}{1-\alpha_1}} L_e^{\frac{1-\alpha}{1-\alpha_1}} A_{ei}. \tag{B.8}$$

Profit maximisation in the energy-intensive and labour-intensive sectors yields the following first-order conditions:

$$L_l = \left(\frac{w}{(1-\alpha)\,p_l \int_0^1 A_{li}^{1-\alpha} x_{li}^{\alpha} di}\right)^{-\frac{1}{\alpha}}, \tag{B.9}$$

$$L_e = \left(\frac{w}{(1-\alpha)\,p_e E^{\alpha_2} \int_0^1 A_{ei}^{1-\alpha_1} x_{ei}^{\alpha_1} di}\right)^{-\frac{1}{\alpha}}, \text{ and} \tag{B.10}$$

$$E = \left(\frac{c_E}{p_e \alpha_2 L_e^{1-\alpha} \int_0^1 A_{ei}^{1-\alpha_1} x_{ei}^{\alpha_1} di}\right)^{\frac{1}{\alpha_2-1}}. \tag{B.11}$$

Plugging the equilibrium quantity of machines (B.5) into (3) yields the production of labour-intensive output:

$$Y_l = L_l A_l \left(\frac{\alpha^2 p_l}{\psi}\right)^{\frac{\alpha}{1-\alpha}}. \tag{B.12}$$

Plugging (B.6) into (B.11) yields the equilibrium quantity of energy:

$$E = \left(\frac{(\alpha_1)^2}{\psi}\right)^{\frac{\alpha_1}{1-\alpha}} \left(\frac{\alpha_2 A_e}{c_E}\right)^{\frac{1-\alpha_1}{1-\alpha}} p_e^{\frac{1}{1-\alpha}} L_e \tag{B.13}$$

Combining (B.13) and (B.6) with (4) yields the production of the energy-intensive good as:

$$Y_e = \left(\frac{(\alpha_1)^2}{\psi}\right)^{\frac{\alpha_1}{1-\alpha}} \left(\frac{\alpha_2 A_e}{c_E}\right)^{\frac{\alpha_2}{1-\alpha}} p_e^{\frac{\alpha}{1-\alpha}} L_e A_e. \tag{B.14}$$

Equilibrium on the labour market implies an identical wage in both sectors. Equating (B.9) and (B.10), together with (B.13), (B.6), and (B.5), yields the relative price:

$$\frac{p_l}{p_e} = \frac{\psi^{\alpha_2} (\alpha_1)^{2\alpha_1} (\alpha_2)^{\alpha_2} A_e^{1-\alpha_1}}{c_E^{\alpha_2} \alpha^{2\alpha} A_l^{1-\alpha}}. \tag{B.15}$$

The relative price (B.1) yields, together with the sectoral production quantities,(B.12) and (B.14), the relative supply in both sectors. Combining relative supply and with relative demands yields the relative employment as:

$$\frac{L_l}{L_e} = \left(\frac{c_E^{\alpha_2} \alpha^{2\alpha}}{\psi^{\alpha_2} (\alpha_1)^{2\alpha_1} (\alpha_2)^{\alpha_2}} \right)^{\epsilon-1} \frac{A_l^{-\varphi}}{A_e^{-\varphi_1}} \tag{B.16}$$

with $\varphi_1 \equiv (1-\alpha_1)(1-\epsilon)$ and $\varphi \equiv (1-\alpha)(1-\epsilon)$.

Finally, the equilibrium prices and quantities can be calculated. The price ratio (B.15), together with the price index (B.2), leads to the equilibrium prices in both sectors:

$$p_l = \frac{\psi^{\alpha_2} \alpha_1^{2\alpha_1} \alpha_2^{\alpha_2} A_e^{1-\alpha_1}}{\left(\left(\alpha^{2\alpha} c_E^{\alpha_2} \right)^{1-\epsilon} A_l^{\varphi} + \left(\psi^{\alpha_2} \alpha_1^{2\alpha_1} \alpha_2^{\alpha_2} \right)^{1-\epsilon} A_e^{\varphi_1} \right)^{\frac{1}{1-\epsilon}}}, \tag{B.17}$$

$$p_e = \frac{\alpha^{2\alpha} c_E^{\alpha_2} A_l^{1-\alpha}}{\left(\left(\alpha^{2\alpha} c_E^{\alpha_2} \right)^{1-\epsilon} A_l^{\varphi} + \left(\psi^{\alpha_2} \alpha_1^{2\alpha_1} \alpha_2^{\alpha_2} \right)^{1-\epsilon} A_e^{\varphi_1} \right)^{\frac{1}{1-\epsilon}}}. \tag{B.18}$$

Combining the prices with input demands yields the equilibrium employment of labour in both sectors

$$L_l = \frac{\left(\psi^{\alpha_2} \alpha_1^{2\alpha_1} \alpha_2^{\alpha_2} \right)^{1-\epsilon} A_e^{\varphi_1}}{\left(\left(\alpha^{2\alpha} c_E^{\alpha_2} \right)^{1-\epsilon} A_l^{\varphi} + \left(\psi^{\alpha_2} \alpha_1^{2\alpha_1} \alpha_2^{\alpha_2} \right)^{1-\epsilon} A_e^{\varphi_1} \right)}, \tag{B.19}$$

$$L_e = \frac{\left(c_E^{\alpha_2} \alpha^{2\alpha} \right)^{1-\epsilon} A_l^{\varphi}}{\left(\left(\alpha^{2\alpha} c_E^{\alpha_2} \right)^{1-\epsilon} A_l^{\varphi} + \left(\psi^{\alpha_2} \alpha_1^{2\alpha_1} \alpha_2^{\alpha_2} \right)^{1-\epsilon} A_e^{\varphi_1} \right)} \tag{B.20}$$

as well as equilibrium energy use in the energy-intensive sector

$$E = \frac{\left(\frac{\alpha_1^2}{\psi} \right)^{\frac{\alpha_1}{1-\alpha}} \alpha_2^{\frac{1-\alpha_1}{1-\alpha}} \alpha^{2\alpha\left(\frac{1}{1-\alpha}-\epsilon+1\right)} c_E^{\alpha_2-1-\epsilon\alpha_2} A_l^{1+\varphi} A_e^{\frac{1-\alpha_1}{1-\alpha}}}{\left(\left(\alpha^{2\alpha} c_E^{\alpha_2} \right)^{1-\epsilon} A_l^{\varphi} + \left(\psi^{\alpha_2} \alpha_1^{2\alpha_1} \alpha_2^{\alpha_2} \right)^{1-\epsilon} A_e^{\varphi_1} \right)^{\frac{1+\varphi}{\varphi}}}. \tag{B.21}$$

Plugging these optimal inputs into (B.12) and (B.14) yields the the equilibrium outputs in the labour- and energy-intensive sector as

$$Y_l = \frac{\alpha^{\frac{2\alpha}{1-\alpha}} \psi^{\frac{\alpha_1(\epsilon\alpha_2-1)}{1-\alpha}} \alpha_1^{\frac{2\alpha_1(1-\epsilon+\epsilon\alpha)}{1-\alpha}} \alpha_2^{\frac{\alpha_2(1-\epsilon-\epsilon\alpha)}{1-\alpha}} A_e^{\frac{1-\alpha_1}{1-\alpha}(\alpha+\varphi)} A_l}{\left(\left(\alpha^{2\alpha} c_E^{\alpha_2}\right)^{1-\epsilon} A_l^{\varphi} + \left(\psi^{\alpha_2} \alpha_1^{2\alpha_1} \alpha_2^{\alpha_2}\right)^{1-\epsilon} A_e^{\varphi_1}\right)^{\frac{\alpha+\varphi}{\varphi}}}, \tag{B.22}$$

$$Y_e = \frac{\left(\frac{\alpha_1^2}{\psi}\right)^{\frac{\alpha_1}{1-\alpha}} \alpha_2^{\frac{\alpha_2}{1-\alpha}} \alpha^{2\alpha\left(\frac{1}{1-\alpha}-\epsilon\right)} c_E^{-\epsilon\alpha_2} A_l^{\alpha+\varphi} A_e^{\frac{1-\alpha_1}{1-\alpha}}}{\left(\left(\alpha^{2\alpha} c_E^{\alpha_2}\right)^{1-\epsilon} A_l^{\varphi} + \left(\psi^{\alpha_2} \alpha_1^{2\alpha_1} \alpha_2^{\alpha_2}\right)^{1-\epsilon} A_e^{\varphi_1}\right)^{\frac{\alpha+\varphi}{\varphi}}}. \tag{B.23}$$

C. Appendix: Equilibrium Profit Ratio and Allocation of Researchers

C.1. Relative Profitability of Research

Since scientists only direct a sector and are randomly allocated to a specific machine variety, the average sectoral productivity is used as defined in (5). Combining (B.7) and (B.8) and taking into account the probabilities of a successful innovation, η_j, the expected firm value (i.e. expected profit) of an innovation in the l-sector, $\Pi_l(t)$, relative to an innovation in the e-sector, $\Pi_e(t)$, is:

$$\frac{\Pi_l(t)}{\Pi_e(t)} = \omega \frac{\eta_l}{\eta_e} \cdot \underbrace{\frac{p_l(t)^{\frac{1}{1-\alpha}}}{p_e(t)^{\frac{1}{1-\alpha_1}}}}_{\text{price effect}} \cdot \underbrace{\frac{L_l(t)}{E(t)^{\frac{\alpha_2}{1-\alpha_1}} L_e(t)^{\frac{1-\alpha}{1-\alpha_1}}}}_{\text{market size effect}} \cdot \underbrace{\frac{A_l(t)}{A_e(t)}}_{\substack{\text{direct} \\ \text{productivity effect}}} \tag{C.1}$$

with $\omega \equiv (1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}}\,(1-\alpha_1)^{-1}\,\alpha_1^{-\frac{1+\alpha_1}{1-\alpha_1}}\,\psi^{\frac{\alpha+\alpha_1}{(1-\alpha)(1-\alpha_1)}}$. Analogously to the Directed Technical Change literature (Acemoglu, 1998, 2002), relative profitability of innovating is affected by a price- and a market size effect. The *price effect* directs innovation in the sector with the higher price. The *market size effect* makes innovations more attractive in the sector, where more factors of production, labour and energy, are employed. Since a larger market size is associated with a lower price for the output of the respective sector, both effects are opposite forces. Finally, the term $A_l(t)/A_e(t)$ captures a *direct productivity effect* as introduced by Acemoglu et al. (2012). This effect directs innovation to the sector that is technologically further advanced and hence follows the concept of "building on the shoulders of giants". In addition to these three forces, the respective probabilities of successful research, η_l and η_e, affect the relative profits.

C.2. Allocation of Researchers

With strong positive (negative) energy price growth, i.e. $\eta_e \gamma (1-\alpha_1)/\alpha_2 < \gamma_{c_E} < (-\eta_l \gamma (1-\alpha)/\alpha_2)$, the direction of the change of relative profit is independent of research.

Proof. For $\epsilon > 1$, it follows with (8), (9), and (10) that,

$$\frac{d\left(\frac{\Pi_l(t)}{\Pi_e(t)}\right)}{dt} = \alpha_2(\epsilon-1)\gamma_{c_E} + \varphi_1 s_e \eta_e \gamma - \varphi s_l \eta_l \gamma > 0 \;\Leftrightarrow \gamma_{c_e} > \eta_e \gamma (1-\alpha_1)/\alpha_2$$

and

$$\frac{d\left(\frac{\Pi_l(t)}{\Pi_e(t)}\right)}{dt} = \alpha_2(\epsilon-1)\gamma_{c_E} + \varphi_1 s_e \eta_e \gamma - \varphi s_l \eta_l \gamma < 0 \;\Leftrightarrow \gamma_{c_e} < -\eta_l \gamma (1-\alpha)/\alpha_2.$$

□

From that it follows that for moderate energy price growth, i.e. $-\eta_l \gamma (1-\alpha)/\alpha_2 \leq \gamma_{c_E} \leq \eta_e \gamma (1-\alpha_1)/\alpha_2$, the direction of the change of relative profit is not independent of research.

Moderate energy price growth

In the case of substitutes ($\epsilon > 1$):

1. From equation (10) and with $s(t) \equiv s_l(t)$ it follows that

$$d\frac{\Pi_{li(t)}}{\Pi_{ei(t)}}/dt \gtreqless 0 \quad \text{if} \quad s(t) \gtreqless \frac{\alpha_2(\epsilon-1)\frac{\gamma_{c_E}}{\gamma} + \eta_l \varphi_1}{\varphi \eta_l + \varphi_1 \eta_e} \equiv s^{**}. \tag{C.2}$$

Proof.

$$d\frac{\Pi_{li(t)}}{\Pi_{ei(t)}}/dt \gtreqless 0 \Leftrightarrow 0 \lesseqgtr \frac{d\frac{\Pi_{li(t)}}{\Pi_{ei(t)}}/dt}{\frac{\Pi_{li(t)}}{\Pi_{ei(t)}}} = \frac{\alpha_2(\epsilon-1)}{c_E(t)}\frac{dc_E(t)}{dt} - \frac{\varphi}{A_l(t)}\frac{dA_l(t)}{dt} + \frac{\varphi_1}{A_e(t)}\frac{dA_e(t)}{dt}.$$

Using equation (8) and (9) yields:

$$0 \lesseqgtr \alpha_2(\epsilon-1)\gamma_c - \varphi s_l(t)\gamma\eta_l + \varphi_1 s_e(t)\gamma\eta_e$$
$$\Leftrightarrow s(t) \gtreqless \frac{\alpha_2(\epsilon-1)\frac{\gamma_{c_E}}{\gamma} + \eta_l \varphi_1}{\varphi \eta_l + \varphi_1 \eta_e} \equiv s^{**}.$$

□

2. At time $t = z$ there exists a unique equilibrium research allocation $s^*(t = z)$ with research directed to sector l (e) only, i.e. $s(t = z) = 1$ ($s(t = z) = 0$), if

$$A(t = z) \equiv \frac{A_e(t = z)^{(1-\alpha_1)}}{A_l(t = z)^{(1-\alpha}} \overset{(>)}{<} \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{\epsilon-1}}.$$

Proof. Using equation (10) yields:

$$\frac{\Pi_l(t)}{\Pi_e(t)} = \kappa \frac{\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}{\eta_e} \frac{A_l(t)^{-\varphi}}{A_e(t)^{-\varphi_1}} \overset{(<)}{>} 1 \Leftrightarrow \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{\epsilon-1}} \overset{(<)}{>} \frac{A_e(t)^{(1-\alpha_1)}}{A_l(t)^{(1-\alpha}} \equiv A(t).$$

□

If $s^*(t = z) \in \{0, 1\}$ is an equilibrium in $t = z$ than it is also an equilibrium in all $t > z$ (follows from Lemma 2 and (C.2)).

3. At time $t = z$ there exist multiple equilibria $s \in [0, 1]$ if

$$A(t = z) = \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{\epsilon-1}}.$$

Proof.

$$\frac{\Pi_l(t)}{\Pi_e(t)} = \kappa \frac{\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}{\eta_e} \frac{A_l(t)^{-\varphi}}{A_e(t)^{-\varphi_1}} = 1 \Leftrightarrow \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{\epsilon-1}} = \frac{A_e(t)^{(1-\alpha_1)}}{A_l(t)^{(1-\alpha}} \equiv A(t).$$

□

If $s^*(t = z) \in (0, 1)$ is an equilibrium in $t = z$ than $s^*(t = z) \in (0, 1)$ is also an equilibrium in $t > z$ if and only if $s^*(t) = s^{**} \ \forall\ t \geq z$. If $s^*(t) \overset{(<)}{>} s^{**}$ there will be research in sector l (e) only in all $t > z$ (follows from Lemma 2 and (C.2)).

In the case of complements ($\epsilon < 1$):

1. From equation (10) follows:

$$d\frac{\Pi_{li(t)}}{\Pi_{ei(t)}}/dt \gtreqless 0 \quad \text{if} \quad s(t = z) \lesseqgtr s^{**}.$$

Proof. See proof for $\epsilon > 1$ and moderate energy price growth. □

2. At time $t = z$ there exists a unique equilibrium research allocation s^* with research directed to sector l (e) only, i.e. $s(t = z) = 1$ ($s(t = z) = 0$), if

$$A(t = z) \overset{(<)}{>} \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{\epsilon-1}}.$$

Proof. See proof for $\epsilon > 1$ and moderate energy price growth. □

With $s^*(t = z) \in \{0, 1\}$ $\left|1 - \frac{\Pi_{li(t)}}{\Pi_{ei(t)}}\right|$ decreases over time and hence there exists a time $\tau > z$ where $\frac{\Pi_{li(t=\tau)}}{\Pi_{ei(t=\tau)}} = 1$ ($\Leftrightarrow A(t = \tau) = \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{\epsilon}-1}$).

3. At time $t = z$ there exist multiple equilibria $s^* \in [0, 1]$, if

$$A(t = z) = \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{\epsilon-1}}.$$

If $s^*(t = z) \in (0, 1)$ is an equilibrium in $t = z$ than $s^*(t = z) \in (0, 1)$ is also an equilibrium in all $t > z$ if and only if $s^*(t) = s^{**}\ \forall\ t \geq z$.
Since $s^* \neq s^{**}$ would result in a(n unrealistic) permanently alternating direction of research, we assume $s^* = s^{**}$ (i.e. the dynamically stable equilibrium) in the case of an inner equilibrium. This is also the technical result for longer patent duration (see Appendix E).

Strong energy price growth

In the case of substitutes ($\epsilon > 1$):

1. At time $t = z$ there exists a unique equilibrium research allocation $s^*(t = z)$ with research directed to sector l (e) only, i.e. $s(t = z) = 1$ ($s(t = z) = 0$), if

$$A(t = z) = \frac{A_e(t = z)^{(1-\alpha_!)}}{A_l(t = z)^{(1-\alpha}} \overset{(>)}{<} \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{\epsilon}-1}.$$

Proof. Using equation (10) yields:

$$\frac{\Pi_l(t)}{\Pi_e(t)} = \kappa \frac{\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}{\eta_e} \frac{A_l(t)^{-\varphi}}{A_e(t)^{-\varphi_1}} \overset{(<)}{>} 1 \Leftrightarrow \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{\epsilon-1}} \overset{(<)}{>} \frac{A_e(t)^{(1-\alpha_!)}}{A_l(t)^{(1-\alpha}} \equiv A(t).$$

□

If $A(t = z) \overset{(>)}{<} \left(\frac{\eta_e}{\kappa \eta_l c_E(t)^{\alpha_2(\epsilon-1)}} \right)^{\frac{1}{\epsilon-1}}$ and with strong positive (negative) energy price growth, $s^*(t = z) = 1$ (0) is an equilibrium in $t = z$ and in all $t > z$ (follows from Lemma 2).

If $A(t = z) \overset{(<}{>} \left(\frac{\eta_e}{\kappa\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{\epsilon-1}}$ and with strong positive (negative) energy price growth, $s^*(t = z) = 0$ (1) is an equilibrium in $t = z$ and since $\left|1 - \frac{\Pi_{li(t)}}{\Pi_{ei(t)}}\right|$ increases over time, there exists a time $\tau > z$, such that $\frac{\Pi_{li(t=\tau)}}{\Pi_{ei(t=\tau)}} = 1$ and $\frac{\Pi_{li(t)}}{\Pi_{ei(t)}} \overset{(<)}{>} 1$ for all $t > \tau$, leading to an equilibrium with research directed to sector l (e) only for all $t > \tau$.

2. At time $t = z$ there exist multiple equilibria $s \in [0, 1]$ if

$$A(t = z) = \left(\frac{\eta_e}{\kappa\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{\epsilon-1}}.$$

Proof.

$$\frac{\Pi_l(t)}{\Pi_e(t)} = \kappa\frac{\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}{\eta_e}\frac{A_l(t)^{-\varphi}}{A_e(t)^{-\varphi_1}} = 1 \Leftrightarrow \left(\frac{\eta_e}{\kappa\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{1-\epsilon}} = \frac{A_e(t)^{(1-\alpha_1)}}{A_l(t)^{(1-\alpha}} \equiv A(t).$$

□

With strong positive (negative) energy price growth, $s^*(t) = 1$ (= 0) is the unique equilibrium in all $t > z$ (follows from Lemma 2).

In the case of complements ($\epsilon < 1$):

1. At time $t = z$ there exists a unique equilibrium research allocation $s^*(t = z)$ with all research directed to sector l (e), i.e. $s(t = z) = 1$ ($s(t = z) = 0$), if

$$A(t = z) \overset{(>)}{<} \left(\frac{\eta_e}{\kappa\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{\epsilon-1}}. \tag{C.3}$$

If $A(t = z) \overset{(>)}{<} \left(\frac{\eta_e}{\kappa\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{\epsilon-1}}$ and with strong positive (negative) energy price growth, $s^*(t = z) = 0$ (1) is an equilibrium in $t = z$ and in all $t > z$, follows from (C.3). If $A(t = z) \overset{(<}{>} \left(\frac{\eta_e}{\kappa\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{\epsilon-1}}$ and with strong positive (negative) energy price growth, $s^*(t = z) = 1$ (0) is a unique equilibrium in $t = z$ and since $\left|1 - \frac{\Pi_{li(t)}}{\Pi_{ei(t)}}\right|$ increases over time, there exists a time $\tau > z$, such that $\frac{\Pi_{li(t=\tau)}}{\Pi_{ei(t=\tau)}} = 1$ and $\frac{\Pi_{li(t)}}{\Pi_{ei(t)}} \overset{(<)}{>} 1$ for all $t > \tau$, leading to an equilibrium with all research directed to sector l (e) for all $t > \tau$.

2. At time $t = z$ there exist multiple equilibria $s^* \in [0, 1]$, if

$$A(t = z) = \left(\frac{\eta_e}{\kappa\eta_l c_E(t)^{\alpha_2(\epsilon-1)}}\right)^{\frac{1}{\epsilon-1}}.$$

With strong positive (negative) energy price growth, $s^*(t) = 0$ $(= 1)$ is the unique equilibrium in all $t > z$ (follows from (C.3)).

D. Appendix: Structural Effect and Efficiency Effect

Proof of Proposition 1

$$\gamma_{\frac{E}{Y}} = [(\alpha_2 S - 1) - S\epsilon\alpha_2]\gamma_{c_E} + [(1-\alpha)S - S(1-\alpha)\epsilon]\gamma_{A_l} + [-(1-\alpha_1)S + S\epsilon(1-\alpha_1)]\gamma_{A_e}$$

Proof. i. Follows from equation (17) with $\gamma_{A_e} > 0$, $\gamma_{A_l} = 0$, and $\gamma_{c_E} = 0$:

$$\begin{aligned} \textit{structural effect} &= (1-\alpha_1)S\epsilon\gamma_{A_e} > 0, \\ \textit{efficiency effect} &= -(1-\alpha_1)S\gamma_{A_e} < 0, \\ \textit{structural effect} + \textit{efficiency effect} \equiv \gamma_{\frac{E}{Y}} &= (\epsilon - 1)(1-\alpha_1)S\gamma_{A_e} \overset{(<)}{>} 0 \Leftrightarrow \epsilon \overset{(<)}{>} 1 \end{aligned}$$

ii. Follows from equation (17) with $\gamma_{A_e} = 0$, $\gamma_{A_l} > 0$, and $\gamma_{c_E} = 0$:

$$\begin{aligned} \textit{structural effect} &= -(1-\alpha)S\epsilon\gamma_{A_l} < 0, \\ \textit{efficiency effect} &= (1-\alpha)S\gamma_{A_l} > 0, \\ \textit{structural effect} + \textit{efficiency effect} \equiv \gamma_{\frac{E}{Y}} &= (1-\epsilon)(1-\alpha)S\gamma_{A_l} \overset{(>)}{<} 0 \Leftrightarrow \epsilon \overset{(<)}{>} 1 \end{aligned}$$

iii. Follows from equation (17) with $\gamma_{A_e} = 0$, $\gamma_{A_l} = 0$, and $\gamma_{c_E} \neq 0$:

$$\begin{aligned} \textit{structural effect} &= -S\epsilon\alpha_2\gamma_{c_E} \overset{(>)}{<} 0 \Leftrightarrow \gamma_{c_E} \overset{(<)}{>} 0, \\ \textit{efficiency effect} &= -(1-\alpha_2 S)\gamma_{c_E} \overset{(>)}{<} 0 \Leftrightarrow \gamma_{c_E} \overset{(<)}{>} 0, \\ \textit{structural effect} + \textit{efficiency effect} \equiv \gamma_{\frac{E}{Y}} &\overset{(>)}{=<} 0 \Leftrightarrow \gamma_{c_E} \overset{(<)}{>} 0. \end{aligned}$$

□

Proof of Proposition 2

Proof. i. Follows from equation (18):

$$\textit{total effect} = \left[\frac{\alpha_2(1-\epsilon)\,A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}} - 1\right]\gamma_{c_E} + \left[\frac{\varphi\,A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma\eta_l < 0$$

$$\Leftrightarrow \gamma_{c_E} > \frac{\varphi A^{1-\epsilon}}{(\alpha_2(1-\epsilon)-1)\,A^{1-\epsilon} - \theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_l \equiv \Lambda_{l,TE} \overset{(<)}{>} 0 \Leftrightarrow \epsilon \overset{(<)}{>} 1,$$

$$\frac{\partial \Lambda_{l,TE}}{\partial A} = -\frac{(1-\epsilon)\varphi A^{-\epsilon}\theta c_E^{\alpha_2(1-\epsilon)}}{\left[(\alpha_2(1-\epsilon)-1)\,A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}\right]^2}\gamma\eta_e < 0.$$

ii. Follows from equation (14):

$$\textit{efficiency effect} = \frac{(\alpha_2-1)A^{1-\epsilon}-\theta c_E^{\alpha_2(1-\epsilon)}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\gamma_{c_E} + \frac{(1-\alpha)A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_l < 0,$$

$$\Leftrightarrow \gamma_{c_E} > \frac{(1-\alpha)A^{1-\epsilon}}{(1-\alpha_2)A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_l \equiv \Lambda_{l,EE} > 0.$$

$$\frac{\partial \Lambda_{l,EE}}{\partial A} = \frac{\varphi A^{-\epsilon}\theta c_E^{\alpha_2(1-\epsilon)}}{\left[(1-\alpha_2)A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}\right]^2}\gamma\eta_l \overset{(>)}{<} 0 \Leftrightarrow \epsilon \overset{(<)}{>} 1.$$

iii. Follows from equation (16):

$$\textit{structural effect} = \frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\epsilon(-\alpha_2\gamma_{c_E}-(1-\alpha)\gamma\eta_l) < 0 \Leftrightarrow \gamma_{c_E} > -\frac{(1-\alpha)}{\alpha_2}\eta_l\gamma.$$

□

Proof of Proposition 3

Proof. i. Follows from equation (18):

$$\textit{total effect} = \left[\frac{\alpha_2(1-\epsilon)\,A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}} - 1\right]\gamma_{c_E} + \left[\frac{-\varphi_1\,A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma\eta_e < 0$$

$$\Leftrightarrow \gamma_{c_E} > \frac{\varphi_1 A^{1-\epsilon}}{(\alpha_2(1-\epsilon)-1)\,A^{1-\epsilon} - \theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_e \equiv \Lambda_{e,TE} \overset{(<)}{>} 0 \Leftrightarrow \epsilon \overset{(<)}{>} 1,$$

$$\frac{\partial \Lambda_{e,TE}}{\partial A} = -\frac{(1-\epsilon)\varphi_1 A^{-\epsilon}\theta c_E^{\alpha_2(1-\epsilon)}}{\left[(\alpha_2(1-\epsilon)-1)A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}\right]^2}\gamma\eta_e < 0.$$

ii. Follows from equation (14):

$$\begin{aligned}
\textit{efficiency effect} = \left[\frac{\alpha_2\, A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}} - 1\right]\gamma_{c_E} + \left[\frac{-(1-\alpha_1)\, A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma\eta_e < 0 \\
\Leftrightarrow \gamma_{c_E} > -\frac{(1-\alpha_1)A^{1-\epsilon}}{(1-\alpha_2)A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_e \equiv \Lambda_{e,EE} < 0,
\end{aligned}$$

$$\frac{\partial \Lambda_{e,EE}}{\partial A} = -\frac{\varphi_1 A^{-\epsilon}\theta c_E^{\alpha_2(1-\epsilon)}}{\left[(1-\alpha_2)A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}\right]^2}\gamma\eta_e \overset{(<)}{>} 0 \Leftrightarrow \epsilon \overset{(<)}{>} 1.$$

iii. Follows from equation (16):

$$\textit{structural effect} = \left[-\frac{\epsilon\alpha_2 A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma_{c_E} + \left[\frac{\epsilon\,(1-\alpha_1)\,A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\right]\gamma\eta_e < 0 \Leftrightarrow \gamma_{c_E} > \frac{(1-\alpha_1)}{\alpha_2}\eta_e\gamma.$$

□

Proof of Proposition 4

Proof. i. Follows from (16):

$$\begin{aligned}
\textit{structural effect} =& -\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\epsilon\alpha_2\gamma_{c_E} - \frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}(1-\alpha)\epsilon\gamma\eta_l\frac{\alpha_2(\epsilon-1)\frac{\gamma_{c_E}}{\gamma}+\eta_e\varphi_1}{\eta_e\varphi_1+\eta_l\varphi} \\
&+ \frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\epsilon(1-\alpha_1)\gamma\eta_e\frac{-\alpha_2(\epsilon-1)\frac{\gamma_{c_E}}{\gamma}+\eta_l\varphi}{\eta_e\varphi_1+\eta_l\varphi} \\
=& -\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\epsilon\alpha_2\gamma_{c_E} + \frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\alpha_2\epsilon\gamma_{c_E}\frac{(1-\alpha)\eta_l+(1-\alpha_1)\eta_e}{\eta_l(1-\alpha)+\eta_e(1-\alpha_1)} \\
&+ \frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\epsilon\gamma\frac{(1-\alpha)\eta_l\eta_e(1-\alpha_1)-(1-\alpha_1)\eta_e\eta_l(1-\alpha)}{\eta_l(1-\alpha)+\eta_e(1-\alpha_1)} \\
=& 0.
\end{aligned}$$

Follows from (14):

$$\begin{aligned}
\textit{efficiency effect} =& \left(\alpha_2 \frac{A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} - 1\right)\gamma_{c_E} + (1-\alpha)\frac{A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_l \frac{\alpha_2(\epsilon-1)\frac{\gamma_{c_E}}{\gamma} + \eta_e\varphi_1}{\eta_e\varphi_1 + \eta_l\varphi} \\
& - (1-\alpha_1)\frac{A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_e \frac{-\alpha_2(\epsilon-1)\frac{\gamma_{c_E}}{\gamma} + \eta_l\varphi}{\eta_e\varphi_1 + \eta_l\varphi} \\
=& \left(\alpha_2 \frac{A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} - 1\right)\gamma_{c_E} + \frac{(-1)(1-\alpha)\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\eta_l\alpha_2\gamma_{c_E}}{\eta_e(1-\alpha_1) + \eta_l(1-\alpha)} \\
& - \frac{(1-\alpha_1)\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\eta_e\alpha_2\gamma_{c_E}}{\eta_e(1-\alpha_1) + \eta_l(1-\alpha)} \\
& + \frac{(1-\alpha)\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_e\eta_l(1-\alpha_1) - (1-\alpha_1)\frac{A^{1-\epsilon}}{A^{1-\epsilon}+\theta c_E^{\alpha_2(1-\epsilon)}}\gamma\eta_e\eta_l(1-\alpha)}{\eta_e(1-\alpha_1) + \eta_l(1-\alpha)} \\
=& \left(\alpha_2 \frac{A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}} - 1\right)\gamma_{c_E} - \frac{A^{1-\epsilon}}{A^{1-\epsilon} + \theta c_E^{\alpha_2(1-\epsilon)}}\alpha_2\gamma_{c_E}\frac{(1-\alpha)\eta_l - (1-\alpha_1)\eta_e}{\eta_e(1-\alpha_1) + \eta_l(1-\alpha)} \\
=& -\gamma_{c_E}.
\end{aligned}$$

ii. As $\gamma_{c_E} > \eta_e\gamma(1-\alpha_1)/\alpha_2 > \Lambda_{e,EE} < 0$ (see Proposition 3), the efficiency effect is negative. For $\gamma_{c_E} > \eta_e\gamma(1-\alpha_1)/\alpha_2$, the structural effect is negative (see Proposition 3). Hence, the total effect must be negative.

iii. As $\gamma_{c_E} < \eta_l\gamma(1-\alpha)/\alpha_2 < \Lambda_{l,EE} > 0$ (see Proposition 2), the efficiency effect is positive. For $\gamma_{c_E} < \eta_l\gamma(1-\alpha)/\alpha_2$, the structural effect is positve (see Proposition 2). Hence, the total effect must be positive.

□

E. Appendix: Direction of Technical Change with infinite-duration Patents

Scientists choose to direct their research at the sector with higher expected firm value (discounted flow of future profits as entrepreneur):

$$E\left[V_{ji}(t=z)\right] = \int_z^\infty E\left[\pi_{ji}(t)\right] exp\left(-\int_z^t \left(1 - E\left[s_j(t)\right]\eta_j\right)dt\right)dt \quad \text{with} j \in \{e, l\}.$$

The expected relative value of firm i in sector j at time $t = z$ comprise current (at time z) and discounted future ($t > z$) expected profits ($E\left[\pi_{ji}(t)\right]$). The expected discount rate ($1 - E[s_j(t)]\eta_j$) depends on the expected research effort in sector j at each time t ($E[s_j(t)]$) and the probability of successful research (η_j). Expected relative firm value at $t = z$ is defined as

$$V(t = z) \equiv \frac{E\left[V_{li}(t = z)\right]}{E\left[V_{ei}(t = z)\right]}.$$

Substitutes (i.e. $\epsilon > 1$):
Since equilibrium research allocation depends crucially on the expected discount rate, the subsequent discussion of research equilibria is structured along three discount rate cases (for special cases see 1. & 3., general case 2.):

1. For $(1 - E[s_j(t)]\eta_j) \to 0$, $V(t = z) \to \frac{\Pi_{li}(t=z)}{\Pi_{ei}(t=z)}$, i.e. relative firm value reduces to current relative firm profits. Results of Appendix C can be applied.

2. For $0 < (1 - E[s_j(t)]\eta_j) < 1$ and since $\frac{\partial E[V_{ji}(t=z)]}{\partial \Pi_{ji}(t)} > 0$, $\frac{\partial E[\Pi_{ji}(t)]}{\partial A_j(t)} > 0$, $\frac{\partial E[\Pi_{ji}(t)]}{\partial A_j(t)} > \frac{\partial E[\Pi_{sector \neq j,i}(t)]}{\partial A_j(t)}$, $\lim_{A_j(t) \to 0} E[\Pi_{ji}(t)] = 0$, $\lim_{A_j(t) \to \infty} E[\Pi_{ji}(t)] = \infty$ for each set of parameters there exists a unique relative technology $(A_l(t = z)/A_e(t = z))^*$ such that $\left.\frac{V_{li}(t=z)|_{s(t)=1}}{V_{ei}(t=z)|_{s(t)=0}}\right|_{\frac{A_l(t=z)}{A_e(t=z)} = \left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*} = 1$. With $\frac{A_l(t=z)}{A_e(t=z)} \overset{(<)}{>} \left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*$, research will take place in the l-sector (e-sector) only. With $\frac{A_l(t=z)}{A_e(t=z)} = \left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*$ there exists a unique equilibrium ($s^{**} \in (0, 1)$) with research directed to both sectors.

   a) With moderate energy price growth, i.e. $-\frac{\eta_l \gamma(1-\alpha)}{\alpha_2} < \gamma_{c_E}(t) < \frac{\eta_h \gamma(1-\alpha_1)}{\alpha_2}$, the expected relative profit (and therefore the expected relative firm value ($V(t)$)) increases (decreases) if research is directed to sector l (e) only (Proof: see Appendix A.3.1). Therefore a research equilibrium $s^* \in \{0, 1\}$ at time z is always a research equilibrium in $t > z$. An inner equilibrium in $t = z$, $s^*(t = z) = s^{**}$, is an inner equilibrium if and only if $s^*(t) = s^{**}\ \forall\ t \geq z$. With $s^*(t = z) \overset{(<)}{>} s^{**}$ research will take place in sector l (e) for all $t > z$ (follows from Appendix A.3.1).

   b) With strong positive (negative) energy price growth and $\frac{A_l(t=z)}{A_e(t=z)} \overset{(<)}{>} \left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*$ research will occur in sector l (e) at $t = z$ and all $t > z$ (follows from Lemma 2 and Appendix A.3.1). If $\frac{A_l(t=z)}{A_e(t=z)} \overset{(>)}{<} \left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*$ and with strong positive (negative) energy price growth, research will at $t = z$ take place in the e-sector (l-sector) only. Since strong positive (negative) energy price growth increases (decreases) $V(t)$, there exists a time $\tau > z$ where $V(\tau) = 1$ and $V(t > \tau) \overset{(<)}{>} 1$, leading to research equilibrium in sector l (e) for all $t > \tau$ (follows from Lemma 2). There are multiple equilibria with $s^*(t = z) \in [0, 1]$ if $\frac{A_l(t=z)}{A_e(t=z)} = \left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*$ and a unique equilibrium with all research in sector l (e) for all $t > z$ in the case of strong positive (negative) energy price growth.

3. For $(1-E[s_j(t)]\eta_j) \to 1$ and moderate energy price growth, $V(t=z) \to 1$ and there exist two equilibria with all research directed to the e- or the l-sector and multiple equilibria with research directed to both sectors (i.e. $s \in (0,1)$). With strong positive (negative) energy price growth there exists a unique equilibrium with all research directed to sector l (e), as $\frac{d\Pi_{e}i(t)}{dt} \to 0$ ($\frac{d\Pi_{l}i(t)}{dt} \to 0$) and therefore $V(t=z) \overset{(0)}{\to \infty}$.

For (plausible) discount rates smaller 1, i.e. $0 \leq (1-E[s_j(t)]\eta_j) < 1$, from 1. and 2. it follows that alternative patent terms do not induce qualitative differences in the research equilibrium at $t=z$. Research takes place in the relatively more advanced sector. Only the value of relative technology thresholds may differ, due to model design.
Research equilibria at $t > z$ are influenced only in so far, as if the direction of research changes over time, i.e. in the case of strong positive (negative) energy price growth and $\frac{A_l(t=z)}{A_e(t=z)} \overset{(>)}{<} \left(\frac{A_l(t=z)}{A_e(t=z)}\right)^*$, the change occurs at an earlier point in time.

Complements (i.e. $\epsilon < 1$):
Results from Appendix C can be applied.