Electric Spot Prices and Wind Forecasts: A dynamic Nordic/Baltic Electricity Market Analysis using Nonlinear Impulse-Response Methodology

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Abstract

This paper uses nonlinear impulse response methodologies to analyse the relationship between the contemporaneous Nordic/Baltic electric spot prices and wind forecasts. The main objective of the analysis is step-ahead spot price movements from wind forecast information. Dynamic impulse response analysis is a technique for analysing the step-ahead characteristics of a nonparametric estimate of the one-step-ahead conditional density. From strictly stationary processes we build a bivariate price/wind semi-parametric density model. The model estimates report a significant negative non-linear covariance between spot prices and wind forecasts movements in the Nordic/Baltic electricity market. The impulse-response analysis shows that the step-ahead contemporaneous co-variance between spot prices and wind forecasts are one important predictor for the spot price mean movements and volatilities. A dynamic nonlinear impulse-response analysis can therefore offer market participants access to dynamic step-ahead spot price mean and volatility strategies.

Classification:

Keywords: Electricity Markets, Nordic/Baltic System Price, Seasonality, Wind Forecasts, Impulse-response functions, Step-Ahead Market Strategies

1 Introduction

This paper studies the characteristics of the bivariate daily spot price and wind forecast movements for the Nordic/Baltic spot electric power market[1]. The prices are fixed based on all participants collected daily purchase and sale requests at 11.00 o'clock. Wind forecasts are available the same morning immediately before 11.00 o'clock and available for all market participants. The system price is the balance price for the aggregated supply and demand graphs; i.e. the price is fixed at market equilibrium. The study is therefore an empirical investigation of the contemporaneous bivariate dynamics of the system prices and the wind forecasts series for the Nordic/Baltic electricity auction market. The empirical effort is mainly data based, owing to limitations of existing theory. Hence, the model class is a bivariate study where simple transformation of squared returns serves as the driving force behind volatility and covariance. The mean and the latent

[1] See Solibakke, 2002.

volatility and covariance specifications will give new and updated information regarding the characteristics of the bivariate spot price and wind forecast relationships.

Using daily spot price and wind forecast movements the emphasis is conditional regularities in mean and volatility dynamics. The dynamic model features together with the latent volatility properties suggest a need for continuous model update. The main emphasis of the paper is therefore twofold. First to establish an optimal bivariate model for spot prices and wind forecasts, and secondly from this bivariate model to establish impulse scenarios for dynamic step-ahead spot price response applications.

The model methodology is a nonparametric time series analysis. The method employs an expansion in hermite functions to approximate the multidimensional conditional density. An appealing feature of this expansion is that it is a nonlinear nonparametric model that directly nests the Gaussian VAR model, the semiparametric VAR model, the Gaussian ARCH model, the semiparametric ARCH model, the Gaussian GARCH model, and the semiparametric GARCH model. The unrestricted SNP expansion model is more general than any of these specific models. The leading term of the series expansion is therefore an established parametric model already known to give a reasonable approximation to the process; higher order terms (hermite functions) capture departures from the model (Robinson, 1983). Moreover, switches can generate simulated sample paths that can be used to compute nonlinear functionals of the density by Monte Carlo integration, notably the nonlinear analogues of the impulse-response mean and volatility profiles used in traditional VAR, ARCH, and GARCH analysis. The SNP model is fitted using conventional maximum likelihood together with a model selection strategy that determines the appropriate order of expansion (Schwarz, 1978). From the daily optimal and bivariate SNP density, the paper applies impulse-response functions for price and wind error shocks. The contribution is a higher understanding of the dynamics of the mean, volatility and covariance processes for the Nordic/Baltic electric power market. The analysis follows mainly three steps. Firstly, the paper addresses seasonality for both series. Deterministic seasonality, scale and trend effects are removed to obtain ergodic stationarity. Deterministic seasonality and (non-)linear trends for the mean and volatility are reported for a 23-years daily series. Secondly, the nonparametric specification seeks consistent mean and volatility specifications applying a non-parametric time series model. Specifically, for the bivariate density, intercept and serial correlation for the mean, and intercept, error shocks, serial correlation, leverage and level effects for the volatility is estimated for the strictly stationary bivariate seasonal and trend adjusted series. Furthermore, hermite function expansions are applied for the approximation of the multidimensional conditional density characteristics. For all expansions, the analysis uses the Schwarz BIC (Schwarz, 1978) information criterion for model specification. An extensive battery of test statistics report model misspecifications. Thirdly and finally, the impulse response analysis shows responses to external shocks for the mean, volatility and covariance. That is, the analysis seeks systematic perturbations influence from the impulse-response curves for the means, the volatilities and the bivariate covariance.

The remainder for this paper is organised as follows. Section 2 introduces the impulse-response functions together with sup-norm confidence bands for all extractable functionals. Section 3 gives a literature overview for the electricity price, the wind forecasts and the non-parametric specification model (SNP). Section 4 defines the data and describes a general adjustment procedure for systematic location and scale effects for stationary bivariate series and a strictly stationary BIC-optimal bivariate model is estimated to obtain the conditional bivariate density. Section 5 performs a post-estimation impulse-response analysis and reports the extended empirical results and establish step-ahead forecasts for dynamic market strategies. Section 6 summarises and concludes.

2 Impulse-Response Functions

The paper applies the methodologies outlined by Gallant et al. (1993) defining one-step ahead forecast for the mean conditioned on the history as $g\left(y_{t-\infty+1},\dots,y_t\right)=E\left(y_{t+1}/\left(y_{t-k}\right)_{k=0}^{\infty}\right)$ in general and $g\left(y_{t-L+1},\dots,y_t\right)=E\left(y_{t+1}/\left(y_{t-k}\right)_{k=0}^{L-1}\right)$ for a Markovian process. We put

$$\begin{aligned} y_j(x) &= E\left(g\left(y_{t-L+j},\dots,y_{t+j}\right)/x_t=x\right) \\ &= E\left(E\left(y_{t+j}/y_{t-L+j},\dots,y_{t+j}\right)/x_t=x\right) \end{aligned}$$

and therefore y_j^i for i = -60, …,60 and j = 0,…,5 where $x=\left(y_{-L+1},\dots,y_0\right)$. The conditional mean profiles $\left\{y_j^i\right\}_{j=1}^{\infty}$ for i = -60,…,60 are the conditional expectations of the trajectories of the one-step-ahead conditional mean. Note that $\left\{y_j^{-10}\right\}_{j=1}^{\infty}$ therefore represents the impulse response to a negative 10% impulse. The impulse responses depend upon the initial x, which reflects the nonlinearity. Moreover, the law of iterated expectations implies that $y_j\left(x\right)=E\left(y_{t+j}/x_t=x\right)$. The sequences $\left\{y_j^i-y_j^0\right\}_{j=0}^{\infty}$, $i=-60,\dots,60$ and $j=0,\dots,5$ therefore represents the effects of the shocks on the trajectories of the process itself. Moreover, a conditional moment profile $E\left[g\left(y_{t+j-J},\dots,y_{t+j}\right)/\left\{y_{t-k}\right\}_{k=0}^{L=1}\right]$, $\left(j=0,1,\dots,5\right)$, the word moment refers to the time-invariant function $g\left(y_{-J},\dots,y_0\right)$.

Similarly, the one-step-ahead variance, also called the volatility, is the one-step ahead forecast of the variance conditioned on history becoming

$$Var\left(y_{t+1}/\left(y_{t-k}\right)_{k=0}^{\infty}\right)=E\left\{\left[y_{t+1}-E\left(y_{t+1}\left\{y_{t-k}\right\}_{k=0}^{\infty}\right)\right]x\left[y_{t+1}-E\left(y_{t+1}/\left\{y_{t-k}\right\}_{k=0}^{\infty}\right)\right]`/\left\{y_{t-k}\right\}_{k=0}^{\infty}\right\}$$ or

$Var\left(y_{t+1} / \left(y_{t-k}\right)_{k=0}^{L-1}\right)$ for a Markovian process. By suitably defining the function g(.), we can measure the effect of shocks on volatility. Now writing

$$\begin{aligned}\Psi_j(x) &= E\left(g\left(y_{t-L+j},\ldots,y_{t+j}\right) / x_t = x\right)\\ &= E\left(Var\left(y_{t+j} / x_{t+j-1}\right) / x_t = x\right)\end{aligned}$$

for *j* =0,…,5 where $x = \left(y_{-L+1},\ldots,y_0\right)$.

$\Psi_j(x)$ is the conditional expectation of the variance matrix *j* steps ahead, conditional on $x_t = x$. Moreover, as for the conditional variance $\left\{\Psi^i_j\right\}_{j=1}^{\infty}$ for *i* = -60%,…,60%. The net effects of perturbations ∂y^i on volatility are assessed by plotting the profiles compared with the baseline $\left\{\Psi^i_j - \Psi^0_j\right\}_{j=1}^{\infty}$, $i = -60,...,60$ and $j = 0,1,...,5$.

Note that the above defined conditional volatility profile, is different from the path described by the *j*-step ahead square error process. Analytical evaluation of the integrals in the definition of a conditional moment profile is intractable. However, evaluation is well suited to Monte Carlo integration. Let $\left\{y^r_j\right\}_{j=1}^{\infty}$, $r = 1,\ldots,R$, be *R* simulated realisations of the process starting from $x_0 = x$. That is, y^r_1 is a random drawing from $f\left(y / x\right)$ with $x = \left(y'_{-L+1},\ldots,y_{-1},y'_0\right)'$; y^r_2 is a random drawing from $f\left(y / x\right)$ with $x = \left(y'_{-L+2},\ldots,y_0,y'_1\right)'$, and so forth. Now applying the invariant function of a stretch of $\left\{y_t\right\}$ of length *j* +1, we get

$$\begin{aligned}g_j(x) &= \int\ldots\int g\left(y_{j-J},\ldots,y_j\right)\left[\prod_{i=0}^{j-1} f\left(y_{i+1} / y_{y-L+1},\ldots,y_i\right)\right]dy_1\ldots dy_j\\ &\doteq (1/R)\sum_{r=1}^{R} g\left(y^r_{j-J},\ldots,y^r_j\right)\end{aligned}$$

with the approximation error tending to zero almost surely as $R \to \infty$, under mild regulatory conditions on *f* and *g*. Sup-norm bands are constructed by bootstrapping, using simulation to consider the sampling variation in the estimation of $f\left(y / x\right)$. That is, changing the seed that generates densities and impulse response samples. The analysis applies 500 samples and 95% confidence intervals. A 95% sup-norm confidence band is an ε-band around the profile $f\left(y / x\right)$ that is just wide enough to contain 95% of the simulated profiles.

3 Background and Literature Review

3.1 The electricity spot price

Several international studies have explored the characteristics and dynamics of Nordic/Baltic spot electricity price series (auction market)[2]. Financial models use historical price series and when assuming stationarity, we can extract reliable characteristics for both the mean and volatility.

Spot electricity prices exhibit high volatility, strong mean reversion[3], frequent spikes and seasonal patterns[4] and differs from region to region (Li and Flynn, 2004). Moreover, Goto and Karolyi (2004) find mean-reversion effect with seasonal changes in volatilities as well as volatility clustering for electricity trading hubs in the U.S., Australia and the Nordic/Baltic market. Chan and Gray (2006) find serial correlation in both the mean and volatility for several electricity markets. Theodorou and Karyampas (2008) studied the less developed and illiquid Greek electricity market. They find mean reversion and the presence of serial correlation in both the mean and the volatility.

A considerable number of models has been proposed in the literature attempting to capture the dynamics of the electricity prices. A class of models includes stochastic models, regime switching models, cointegration analysis, mean-reverting models and other empirical models[5]. These models fail to capture the full volatility dynamics of electricity prices as well as, the price and volatility interrelationships. Another class of models introduces univariate Generalized Autoregressive Conditional Heteroscedasticity (GARCH) conditional volatility models, as well as other variations of GARCH modelling, such as EGARCH and TGARCH[6]. These models capture the price and volatility dynamics of electricity prices, as well as price shock transmissions. However, univariate models fail to capture the full dynamics that exist in electricity market. Modelling mean and volatility interrelationships between different time series requires the extension of econometric modelling to the multivariate level. The use of VAR models and Multivariate GARCH (MGARCH) models extends modelling capturing inter-dynamics between series. That is, the dependence developed between contemporaneous mean and volatility with respect to the past mean, volatility and shocks, of other time series. Finally, Knittel and Roberts (2005) find an inverse leverage effect for electricity prices in the US. Other studies have found similar results[7]. For the purpose of this study, we build and extend the work of Solibakke (2002) and follow the

[2] See for example Kristiansen (2014) Goto and Karolyi (2004), Bystrøm (2003) and Solibakke (2002).
[3] See e.g. Lucia and Schwartz, 2002 and Geman and Roncoroni, 2006.
[4] See Higgs and Worthington (2008), Huisman and Mahieu ((2003) and Thomas et al., (2011).
[5] See De Vany and Walls (1999), Higgs and Worthington (2008), Huisman and Mahieu (2003), Huisman and Kilic (2013), Haldrup and Nilsen (2006), Knittel (2005), Li and Flynn (2004), Lindstrom and Regland (2012), Mount, Ning and Cai (2006), Robinson (2000), Robinson and Baniak (2002), Rubin and Babcock (2011), Tashpulatov (2013), and Weron (2006, 2008).
[6] See Chan and Gray (2006), Escribano, Pena and Villaplana (2011), Habell, Marathe and Shawky (2004), Higgs and Worthington (2005), Koopman, Ooms and Carnero (2007) and Solibakke (2002).
[7] See i.e. Weron (2006, 2008), Harris (2006), Geman and Roncoroni (2006), Koopman et al. (2007) and Pilipovic (2007).

methodology of Gallant and Tauchen (1993, 2014). For bivariate prices and wind forecast, seasonalities and trends are extracted and strictly stationary time series semi non-parametric model (SNP) model is estimated (Gallant and Tauchen, 2010). We perform a post-estimation analysis for the non-linear impulse-response methodology from Section 2 above.

3.2 Wind Forecasts and Spot Price Movements and Volatility

The share of wind power in electricity generation has been rapidly increasing in the Nordic/Baltic market. In April 2013 wind power generation was 973,520 MwH and in April 2017 2,930,921 MwH. That is, over a four year period the monthly average production has tripled. Wind power has nearly zero marginal production costs and is often subsidized (Skytte, 1999 and Morthorst, 2003). Wind power generation is therefore dispatched prior to other generators, leaving residual demand to other technologies (merit order effect)[8]. In summary therefore, high level of wind generation is expected to decrease electricity spot prices, suggesting a natural negative correlation between wind generation and spot price movements. As shown by Giabardo et al. (2009) estimated future wind power generation appears as a stochastic threshold in the supply function.

Considerable less attention has been given to wind generation and price volatility. Since wind generation originates from meteorological conditions, the supply of wind power is easily classified as exogenous impulses. For periods with shifting wind power generation the volatility of spot electricity prices will most likely increase, dependent on the flexibility of other generators. The Nordic/Baltic market with abundant hydro resources, have a natural tool to cope with undirected variations in wind output, reducing spot price volatility. The impact of wind generation on electricity prices and volatility will create speculation opportunities and of course impact investment decisions. As wind power has tripled and become more competitive, it has raised more challenges for market operators. Hence, more effort has been made on modelling the displacement of technologies brought by merit order effect and the incentives to invest in different technologies under the envisaged growth of RES[9] use. For electricity mean prices, Forrest and Mac Gill (2013) show that wind penetrations in the Australian electricity market are negatively correlated with the wholesale price and have grater effects at high levels of demand. This point of view is shared with Ciarreta et al. (2014) for the case of Spain, as well as with Traber and Kenfert (2011 for the case of Germany.

For the case of price volatility, the impact on spot price stability caused by wind deployment, Green and Vasilakos (2010), Steggals et al. (2011), Woo et al. (2011), Jacobsen and Zvingilaite (2010), and Twomey and Neuhoff (2010),

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[8] An additional factor is congestion in transmission systems potentially leading to area prices (EWEA, 2010).
[9] Renewable Energy Sources (RES). The European Commission (EC) aims at raising the share of RES in energy consumption to 20% by 2020 (EC, 2009) and at least 27% by 20130 (EC, 2014).

have found increased price variations when electricity markets rely on a large share of intermittent generation. These works are all interpreted as giving support for a threat to overall electricity supply resulting from fluctuations of wind output. For the case of Denmark West bidding area, Jònsson et al. (2010) illustrate using non-parametric regression, not only a discontinuous effect on price reduction, but also diminishing intraday price variations caused by wind penetration. For the Nordic/Baltic region some additional work has been dedicated to the implementation and integration of wind power, from the perspectives of macroeconomics (Sperling et al., 2010), geographical aggregation (Østergaard, 2008) and end-user demand responsiveness (Grohnheit et al., 2011). Munkgaard and Morthorst (2008) recognized that risk-averse investor would be reluctant to invest in wind installation in Denmark after a high feed-in tariff scheme was replaced by a new tariff scheme aiming at a smooth transition from the guaranteed price to the market price for wind producers. However, none of these papers has explicitly quantified the impacts of large wind penetration on the day-ahead market or examined the variations of markets signals facing wind intermittency. Moreover, the bivariate spot price and wind forecast movements, despite its importance, are to my best knowledge, not performed for the Nordic/Baltic region. Therefore, the current paper is to fill the gap in the literature to conduct an econometric analysis on the day-ahead performance in relation to wind deployment.

3.3 The SNP nonparametric mean and volatility methodology

Non-linear stochastic models will in our study imply conditional models and so-called ARMA-GARCH methodology. Autoregressive and moving average (ARMA) is terms applied to the structure of the conditional mean, while generalized autoregressive conditional heteroscedasticity (GARCH) is terms applied to the structure of the conditional volatility. ARMA models can be studied in details in for example Mills (1990), while ARCH specifications was first studied by Engle (1982), and moved further on by Bollerslev (1986) who specified the Generalized ARCH or GARCH. The development to GARCH from ARCH was done preliminary owing to the number of lags in the ARCH specification[10]. ARCH/GARCH specifies the volatility as a function of historic price changes and volatility. In the international finance literature quite a number of studies has shown the use of results from these pioneer works. See for example Bollerslev et al. (1987, 1992), Engle et al. (1986, 1993), Nelson (1991) and deLima (1995a, 1995b). For a comprehensive introduction to ARCH models and applications in finance see Gouriéroux (1997).

Ding et al. (1993) extends the symmetric GARCH model into asymmetric GARCH and the truncated GARCH (GJR) is described by Glosten et al (1993). M(aximum) L(ikelihood) estimates of the GARCH-in-Mean model can be obtained by maximising the likelihood function. Note however, that the information matrix is no longer block diagonal, so that all the

[10] Gallant & Tauchen (1997) find 18 (!) ARCH-lags for time series retrieved from the US financial market.

parameters must be estimated simultaneously. This requires an iterative solution technique[11], also known as non-linear optimisation.

SNP stands for SemiNonParametric, to suggest that it lies halfway between parametric and nonparametric procedures. The leading term of the series expansion is an established parametric model known to give a reasonable approximation to the process; higher order terms capture departures from that model. With this structure, the SNP approach does not suffer from the curse of dimensionality to the same extent as kernels and splines. In regions where data are sparse, the leading term helps to fill in smoothly between data points. Where data are plentiful, the higher order terms accommodate deviations from the leading term and fits are comparable to the kernel estimates proposed by Robinson (1983). The theoretical foundation of the method is the Hermite series expansion, which for time series data is particularly attractive on the basis of both modelling and computational considerations. In terms of modelling, the Gaussian component of the Hermite expansion makes it easy to subsume into the leading term familiar time series models, including VAR, ARCH, and GARCH models (Engle, 1982; Bollerslev, 1986). These models are generally considered to give excellent first approximations in a wide variety of applications. In terms of computation, a Hermite density is easy to evaluate and differentiate. Also, its moments are easy to evaluate because they correspond to higher moments of the normal, which can be computed using standard recursions. Finally, a Hermite density turns out to be very practicable to sample from, which facilitates simulation. The form is:

$$\begin{aligned}\sum x_{t-1} &= R_0 \cdot R_0' + \sum_{i=1}^{L_g} Q_i \sum{}_{x_{t-1-i}} Q_i' + \sum_{i=1}^{L_r} P_i \left(y_{t-i} - \mu_{x_{t-1-i}}\right)\left(y_{t-i} - \mu_{x_{t-1-i}}\right)' P_i' \\ &+ \sum_{i=1}^{L_v} \max\left[0, V_i\left(y_{t-i} - \mu_{x_{t-1-i}}\right)\right] \max\left[0, V_i\left(y_{t-i} - \mu_{x_{t-1-i}}\right)\right]' + \sum_{i=1}^{L_w} W_i \cdot x_{(1),t-i} \cdot x_{(1),t-i}' \cdot W_i'\end{aligned}$$

where R_0 is an upper triangular matrix. The matrices P_i, Q_i, V_i, and W_i can be scalar, diagonal, or full M by M matrices. The notation $x_{(1),t-i}$ indicates that only the first column of x_{t-i} enters the computation. The max(0, x) function is applied elementwise.

4 Empirical Data, Deterministic Adjustments and SNP Projections

4.1 Empirical Data and Deterministic Adjustments

The study uses daily prices of the so-called system price and daily wind forecasts for the Nordic/Baltic s pot market for electric power spanning the period from January 2013 to January 2017. The daily prices are the average prices for 24 hours auction (system) prices. Wind forecasts are reported daily hour by hour and aggregated to 24 hour intervals. When changing to winter (summer) time the price and wind one hour is lost (doubled) and is carefully handled in the two series.

[11] The technique is available from version GAUSS 3.2.1.

We adjust for systematic location and scale effects (Gallant, Rossi and Tauchen, 1992) in both returns and volatility. Let ϖ denote the variable to be adjusted. Initially, the regression to the mean equation $\varpi = x \cdot \beta + u$ is fitted, where x consists of calendar variables as are most convenient for the time series and contains parameters for trends, day of week and week number dummies, calendar day separation variable, and sub-periods. To the residuals, $\hat{u}$, the variance equation model $\hat{u}^2 = x \cdot \gamma + \varepsilon$ is estimated. Next $\frac{\hat{u}^2}{\sqrt{e^{x \cdot \hat{\gamma}}}}$ is formed, leaving a series with mean zero and (approximately) unit variance given x. Lastly, the series $\hat{\varpi} = a + b \cdot (\frac{\hat{u}}{\sqrt{e^{x\hat{\gamma}}}})$ is taken as the adjusted series, where a and b are chosen so that $\frac{1}{T} \cdot \sum_{i=1}^{T} \hat{\varpi}_i = \frac{1}{T} \cdot \sum_{i=1}^{T} \varpi_i$ and $\frac{1}{T-1} \cdot \sum_{i=1}^{T} (\hat{\varpi}_i - \overline{\varpi})^2 = \frac{1}{T-1} \cdot \sum_{i=1}^{T} (\hat{u}_i - \overline{u})^2$. The purpose of the final location and scale transformation is to aid interpretation. In particular, the unit of measurement of the adjusted series is the same as that of the original series. Table 1 reports the deterministic seasonal, scale and trend effects in the two series.

The characteristics of the adjusted system price and wind prognosis series are reported in Table 2. For the system price movements, the mean is close to zero. The standard deviation seems high relative to other commodity markets, and must be attributed to the close to non-storable features of an electricity market dominated by hydro power stations. The Cramer-von-Mises test statistic[12] confirms non-normal series densities. Serial correlation in the mean (Q[13]) and volatility (Q^2 and ARCH (Engle, 1982)). The series is confirmed stationary with both the KPSS and Augmented DF test statistics. The BDS-Z-statistic (Brock and Deckert, 1988 and Scheinkman, 1990) (ε=1) suggests significant data dependence. For the wind forecast movements, the mean is close to zero. The wind forecasts report a high standard deviation and is highly volatile confirmed by the maximum and minimum numbers of the series. The series show non-normal features and serial correlation. The KPSS and Augmented DF test statics confirm stationarity. Volatility clustering and general data dependence exist but is clearly lower than for the spot price series. Finally, for both series, the RESET(12;6) (Ramsey, 1969) suggest non-linearity in the mean. We report paths and distributions for the seasonal adjusted system price and wind forecast movements in Figure 1 and 2. The paths seem stationary (around zero). The kernel distributions are not normal and closer to student's *t* or logistic. Figure 3 reports a bivariate scatterplot of the adjusted spot price and wind forecasts series. A linear trend function in Figure 3 reads $y = -0.1071x + 0 \ ; t = -24.5 \ and \ R^2 = 29.1\%$. The negative t and an R^2 of 29.1% suggests a significant negative correlation between spot prices and wind forecasts of approximately -0.56 ($\sqrt{29.1\%}$). This significant negative relationship between spot and wind will be analysed using a formalised impulse-response model approach.

[12] The Cramer-von-Mises test statistic is a procedure to test the null that a sample comes from a population in which the variable is distributed according to a normal distribution.
[13] See Box & Jenkins, 1976 and Ljung and Box, 1978.

Table 1. Seasonal characteristics of the Mean and Volatility (2013-2017) for the Raw System Price and Wind Forecast Daily Movements

Returns dy=(ln(S_t / S_{t-1})

Var	Coeff	t-Stat
INTR	0.0971	0.2554
MON	13.9304	18.7100
FRI	-2.6080	3.5093
SAT	-8.7229	11.7360
SUN	-3.0678	4.1275
Eastern H	-5.1045	2.3889
General H	-9.8865	2.7841
Week-26	13.1580	3.7054
Week-27	7.5700	1.8958
Week-28	10.2947	2.5781
Week-29	9.3663	2.3456
Week-30	10.5704	2.6472

Volatility σ_y=(ln (res^2))

Var	Coeff	t-Stat
INTR	1.6031	8.9481
MON	1.3115	7.8333
SAT	0.7488	4.4723
School	-0.5403	2.4499
Week-5	-0.8878	2.2510
Week-13	-0.8826	2.2376
Week-27	-0.7815	1.7734
Week-35	-1.1827	2.6931
Week-37	-1.3329	3.0331
Week-38	-0.9040	2.0570
Week-42	-1.4822	3.3726
Week-46	-2.4165	5.4976
Week-47	-1.1622	2.6440
Trend	3.7464	4.6527
Squared T	-3.0578	3.9200

Wind Forecast dW=(ln(W_t / W_{t-1})

Var	Coeff	t-Stat
INTR	-0.0705	0.4399
SUN	0.7813	1.8558
Other H	-2.0082	1.7331

Volatility σ_W=(ln (res^2))

Var	Coeff	t-Stat
INTR	3.2568	19.9410
Other H	0.9033	1.9807
Week-5	-1.6385	4.4486
Week-52	-1.4220	3.2312
Trend	-3.9191	5.2165
Squared T	2.6643	3.6657

Exogenous Variables (only close to significant coefficients (10%) are reported):
INTR=Constant; MON=Monday; FRI=Friday, SAT=Saturday; SUN=Sunday;
Eastern H = Eastern Holidays; Other H=Other Holidays; General H=General Holidays;
Week-1-Week-52; Tend=linear trend; Squared T=Squared trend;

Table 2. Statistics for NordPool Spot-System electricity Price and Logarithmic Wind Prognosis Movements

Spot/	Mean /	Median	Maximum /	Moment	Quantile	Quantile	Cramer-von	Serial dependence	
System	Mode	Std.dev.	Minimum	Kurt/Skew	Kurt/Skew	Normal	Mises	Q(12)	$Q^2(12)$
Price	-0.01053	0.69461	58.5853	6.38381	0.22475	5.0829	2.84401	70.3650	219.19
		12.30240	-90.7019	-0.73879	-0.08154	{0.0788}	{0.0000}	{0.0000}	{0.0000}
	BDS-Z-statistics (ε=1)				KPSS (Stationary)		Augmented	ARCH	RESET
	m=2	m=3	m=4	m=5	Intercept	I&Trend	DF-test	(12)	(12,6)
	9.0586	12.0101	13.3772	13.9937	0.07066	0.05971	-23.84982	146.98888	13.38060
	{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.2218}	{0.1509}	{0.0000}	{0.0000}	{0.0374}
Wind	Mean /	Median	Maximum /	Moment	Quantile	Quantile	Cramer-von	Serial dependence	
Prognosis	Mode	Std.dev.	Minimum	Kurt/Skew	Kurt/Skew	Normal	Mises	Q(12)	$Q^2(12)$
	0.00812	-0.18648	27.2324	0.98531	0.12473	2.4569	0.34374	119.036	52.11661
		5.86777	-25.0394	0.20419	0.07368	{0.2927}	{0.0001}	{0.0000}	{0.0000}
	BDS-Z-statistics (ε=1)				KPSS (Stationary)		Augmented	ARCH	RESET
	m=2	m=3	m=4	m=5	Intercept	I&Trend	DF-test	(12)	(12,6)
	1.2702	3.9674	5.3578	5.7620	0.18534	0.08421	-21.22916	42.85096	35.43575
	{0.2040}	{0.0001}	{0.0000}	{0.0000}	{0.1818}	{0.1009}	{0.0000}	{0.0000}	{0.0000}

Table 4. Residual Statistics for Spot System Price and Wind Prognosis Movements

Spot	Mean /	Median /	Maximum /	Moment	Quantile	Quantile	Cramer-von	Serial dependence	
System	Mode	Std.dev.	Minimum	Kurt/Skew	Kurt/Skew	Normal	Mises	Q(12)	$Q^2(12)$
Price	0.01072	0.05393	4.61665	1.08726	0.08778	1.17874	0.53491	8.7402	14.902916
		1.00783	-4.62070	-0.20338	-0.05106	{0.5547}	{0.0000}	{0.7249}	{0.2468}
	BDS-statistic (ε=1)				ARCH	RESET	Joint		
	m=2	m=3	m=4	m=5	(12)	(12;6)	Bias		
	-1.97196	-1.14503	-1.14631	-1.47460	14.70202	13.72420	8.61220		
	{0.0486}	{0.2522}	{0.2517}	{0.1403}	{0.2581}	{0.0329}	{0.0349}		
Wind	Mean /	Median /	Maximum /	Moment	Quantile	Quantile	Cramer-von	Serial dependence	
Prognosis	Mode	Std.dev.	Minimum	Kurt/Skew	Kurt/Skew	Normal	Mises	Q(12)	$Q^2(12)$
	-0.03698	-0.05746	4.45881	3.64818	0.14563	1.69690	0.66891	13.6379	12.95043
		0.99214	-7.10531	0.04974	0.03499	{0.4281}	{0.0000}	{0.3244}	{0.3727}
	BDS-statistic (ε=1)				ARCH	RESET	Joint		
	m=2	m=3	m=4	m=5	(12)	(12;6)	Bias		
	-0.11339	0.68555	0.67333	0.53754	11.84137	4.89385	2.58156		
	{0.9097}	{0.4930}	{0.5007}	{0.5909}	{0.4585}	{0.5575}	{0.4607}		

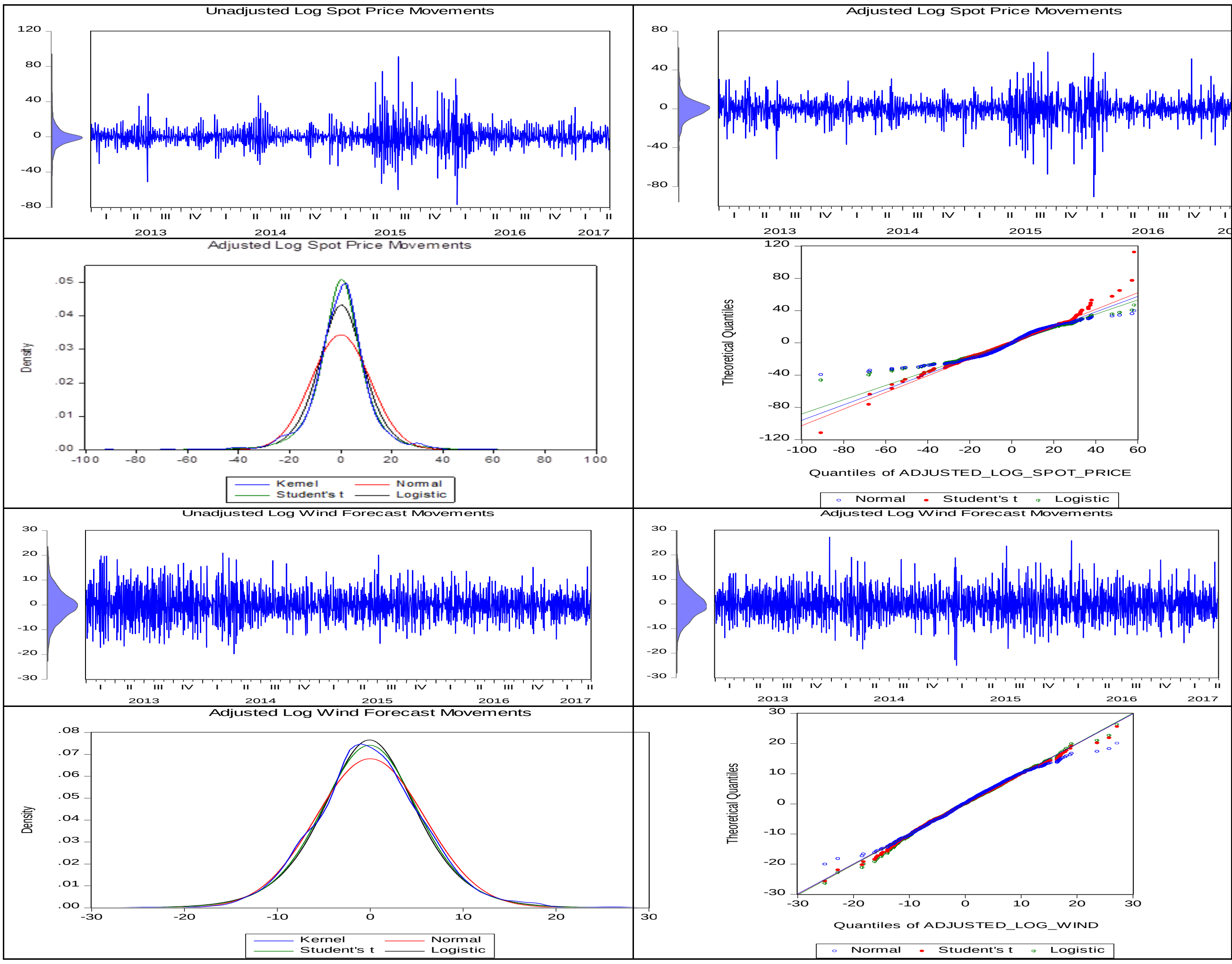
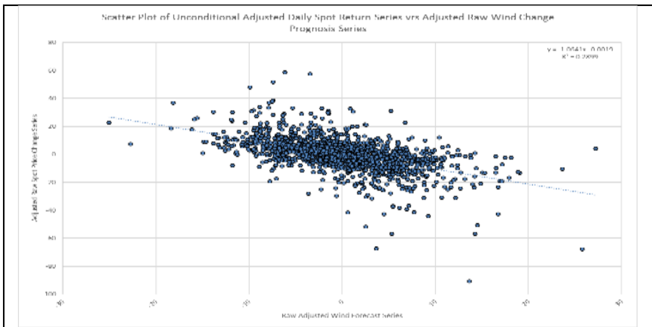


Figure 1 and 2 Unadjusted and Adjusted Spot price and Wind Forecast Movements, Densities and QQ-plots

4.2 The SNP Density Projection (the model)

The estimation method is termed SNP, which stands for semi-nonparametric, to suggest that it lies halfway between parametric and nonparametric procedures. It is based on an expansion in Hermite functions, for estimation of the conditional density. Since the conditional density completely characterizes the processes, it is thus naturally viewed as the fundamental statistical object of interest. The leading term of the bivariate system price and wind forecast change series expansion is an established parametric model known to give a reasonable approximation to the process (VAR-GARCH); higher order terms capture departures from that model (Hermite functions). The SNP model is fitted using conventional maximum likelihood together with the BIC (Schwarz, 1978) model selection strategy. The Schwarz Bayes information criterion (Schwarz, 1978) is computed as $BIC = s_n\left(\hat{\theta}\right)+\left(\frac{1}{2}\right)\left(\frac{p_p}{n}\right)\log\left(n\right)$ with small values of the criterion preferred. The ML estimates[14] of the parameters for the SNP model specification on the adjusted bivariate time series are reported in Table 3. Firstly, for the mean, the intercept is insignificant and the serial correlations (*B[1,x]*) for almost all lags are significant implying dependence up to 7 days (one week). We find strong positive autocorrelation[15] and the 7-days week structure in the serial correlation seems a natural candidate for correlation structure. Secondly, the conditional variance coefficients show an obvious correlation structure. Conditional heteroscedasticity is present. Asymmetry (*V*) is low but the level effects are present for the wind forecasts (*W*). Finally, the hermite functions coefficients (a_0[1]- a_0[8]), capturing departures from the classical normally distributed model, are BIC preferred up to four polynomial expansions. A scatter plot from a long 250 *k* model simulation is reported in Figure 4. The simulation seems in accordance with the original data from Figure 3. The bivariate SNP model gives access to an extended information set. A conditional bivariate scatter plot for spot prices and wind forecasts are reported in Figure 5. Figure 5 includes a simple linear trend function. The equation becomes $y = -0.0865x + 0.1771$; $t = -26.7$; $R^2 = 32.99\%$, where *y* is conditional spot prices and *x* is conditional wind forecasts. The influence from wind forecasts are significantly (t = -26.7) present in the seasonally adjusted spot prices.



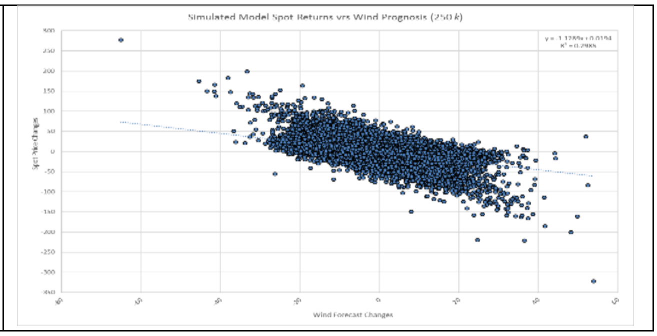


Figure 3 qnd 4 Scatter Plot of the ADJUSTED Spot Price and Wind Forecast Movements ***(1.5 k and simulated 250 k)***

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[14] Based on Likelihood Ratio Test statistics (LRT) the student-t log-likelihood function is strongly preferred to a normal likelihood function.

[15] The serial correlation may stem from the strong day effects found in the adjustment procedure. Our procedure seems not to remove all systematic seasonal effects.

Table 3 Bivariate SNP model: System Price and Wind Forecast Movements

Hermite Polynomials:

Descriptor		theta (q)	std.error	t-statistics
a0[1]	01	0.003650	0.013460	0.271200
a0[2]	02	0.006930	0.045740	0.151410
a0[3]	03	0.020350	0.014580	1.395410
a0[4]	04	0.045730	0.013890	3.293160
a0[5]	10	0.015520	0.013410	1.157010
a0[6]	11	0.075410	0.041690	1.808600
a0[7]	12	0.014110	0.013560	1.040860
a0[8]	13	0.006130	0.014560	0.420890
a0[9]	20	-0.061270	0.031000	-1.976550
a0[10]	21	0.009340	0.014770	0.632550
a0[11]	22	0.074460	0.019310	3.856170
a0[12]	30	-0.035190	0.013160	-2.674140
a0[13]	31	-0.077760	0.015550	-5.000740
a0[14]	40	0.097900	0.017700	5.530400

SNP-model Specification	71f1f14000
Schwarz (BIC) criterion:	2.58437
-2 ln like	-3863.7969
Largest eigenvalue of mean function m	0.8066450
Largest eigenvalue of variance functio:	0.9755410

Bivariate Mean Equation:

B(1,1)	-0.08566	0.02711	-3.15912
B(2,1)	-0.07452	0.02662	-2.79971
B(1,2)	-0.03662	0.02054	-1.78293
B(2,2)	-0.16742	0.02761	-6.06407
B(1,3)	-0.19678	0.0241	-8.1656
B(2,3)	-0.12116	0.02451	-4.94372
B(1,4)	-0.0876	0.01883	-4.65134
B(2,4)	-0.27569	0.02522	-10.93305
B(1,5)	-0.1116	0.02619	-4.26184
B(2,5)	-0.0857	0.02535	-3.38087
B(1,6)	-0.06927	0.01936	-3.57859
B(2,6)	-0.17957	0.02536	-7.08009
B(1,7)	-0.10934	0.02412	-4.53263
B(2,7)	-0.08699	0.02546	-3.41674
B(1,8)	-0.04989	0.02081	-2.39682
B(2,8)	-0.15402	0.02552	-6.03628
B(1,9)	-0.07901	0.02403	-3.28807
B(2,9)	-0.02042	0.02467	-0.8276
B(1,10)	-0.00458	0.02076	-0.22045
B(2,10)	-0.11758	0.0251	-4.68487
B(1,11)	-0.00852	0.02236	-0.38124
B(2,11)	-0.04396	0.0251	-1.75105
B(1,12)	-0.07347	0.02028	-3.62268
B(2,12)	-0.07348	0.02289	-3.21028
B(1,13)	0.05547	0.0234	2.3709
B(2,13)	-0.10655	0.02314	-4.60529
B(1,14)	-0.12123	0.01922	-6.30833
B(2,14)	0.01929	0.02336	0.82597

Bivariate Volatility Equation:

R0[1]	-0.01099	0.26732	-0.04111
R0[2]	0.0472	0.06662	0.70847
R0[3]	0.41671	0.05854	7.11852
P(1,1) f	0.30213	0.03693	8.182
P(2,1) f	0.02454	0.04954	0.49522
P(1,2) f	-0.03601	0.03339	-1.07851
P(2,2) f	0.11589	0.06099	1.9003
Q(1,1) f	0.94536	0.01392	67.92248
Q(2,1) f	-0.06081	0.02073	-2.93342
Q(1,2) f	0.03061	0.02986	1.02518
Q(2,2) f	0.80747	0.04446	18.16252
V(1,1) f	-0.07647	0.12246	-0.62444
V(2,1) f	0.12017	0.06385	1.882
V(1,2) f	-0.08933	0.0582	-1.53472
V(2,2) f	0.40645	0.04965	8.18706
W(1,1) f	0.46589	0.06356	7.32965
W(2,1) f	-0.19548	0.04687	-4.17062
W(1,2) f	-0.2029	0.04856	-4.17829
W(2,2) f	0.06024	0.05947	1.01296

For the conditional trend function the constant is only marginal significant. The conditional variance is reported in Figure 6 together with a calculation of moving averages with lags of 4 and 15 (m = 4, 15). From the plots we can easily observe that the moving average model is more suitable for the spot price. Moreover, the bivariate dimension gives access to measures of covariance. Figure 6 also reports the correlation pattern between spot prices and wind forecasts. The correlation mean is about -0.56 but it moves between a correlation between 0.3 and 0.9 on a daily basis for the four years' period. Finally, Figure 7 reports asymmetric volatility represented by the conditional variance/covariance functions. There are only small differences between negative and positive price changes in the conditional variance-covariance processes. The covariance process is clearly symmetric. Figure 7 also report the one-day-ahead conditional bivariate density together with a quadrature plot for spot prices and wind forecasts. Both plots indicate daily changing symmetric variance/covariance matrices. The largest eigenvalue of the conditional variance function P & Q companion matrix is 0.978 suggesting low persistence in the bivariate variance-covariance processes.

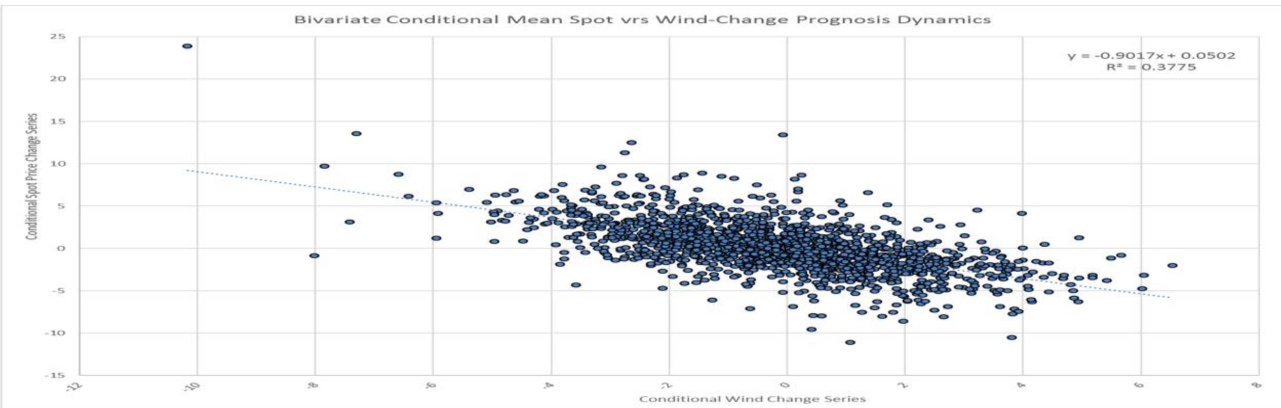

Figure 5. Scatter Plot of the Conditional Spot Price and Wind Forecast Movements

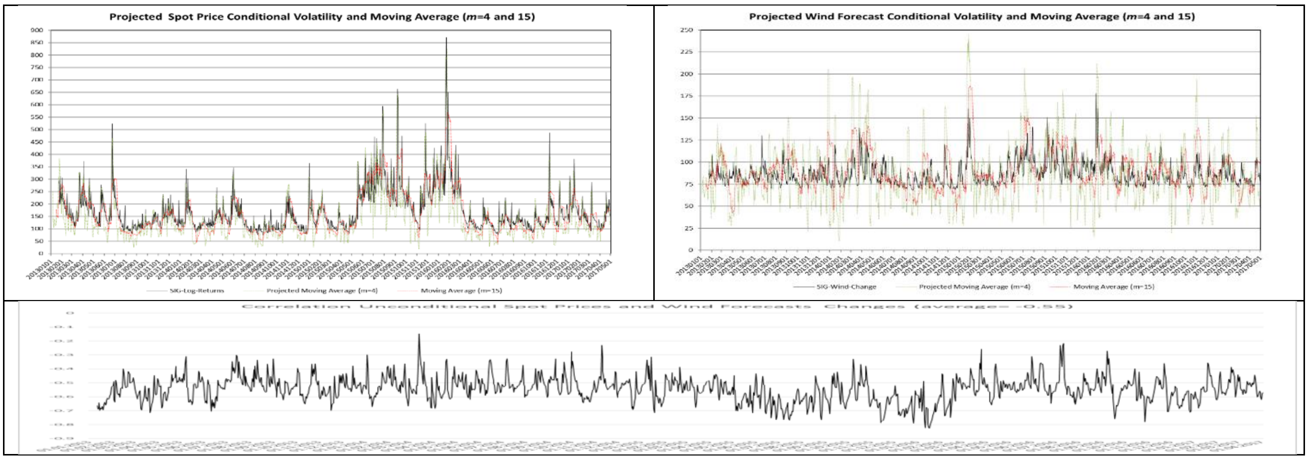

Figure 6. Conditional Spot Price and Wind Forecast Co-Variance with associated Moving Averages (m=4 and m=15)

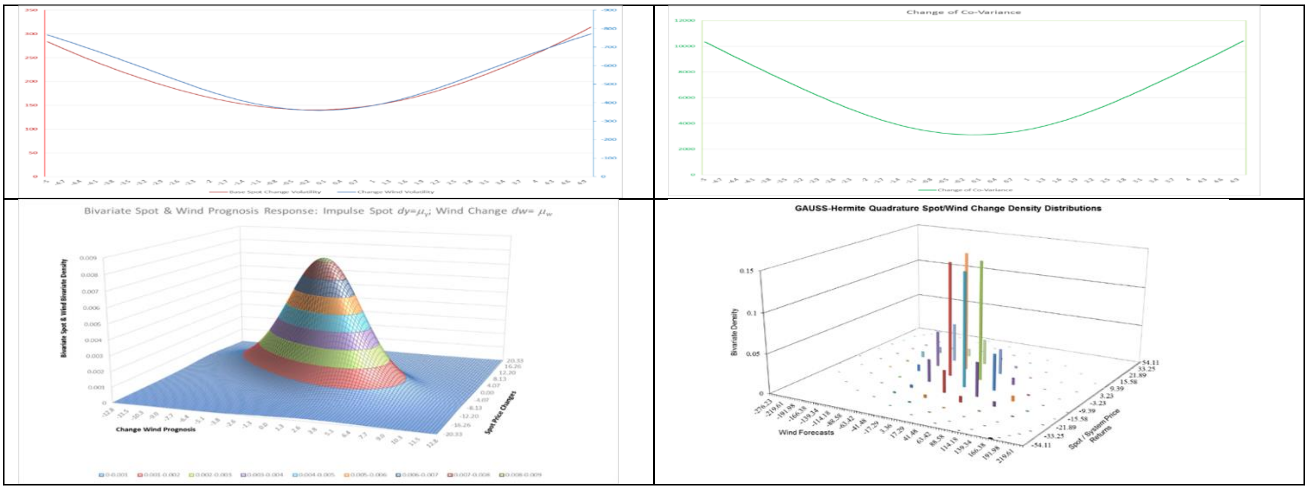

Figure 7. Conditional Asymmetric Volatility and Covariance Plots

From the standardised residuals of the SNP semiparametric model the residuals establish a basis for model misspecification tests. Table 4 reports test statistics together with distributional properties. For both series, the mean for the standardised residuals is close to zero and the standard deviation is close to one (N(0,1)). However, the Cramer-von-Mises test statistics suggest deviations from standard normal distributions for the residuals. The twelfth order Ljung and Box (1978) statistic for the standardised residuals (Q), squared standardised residuals (Q^2), the ARCH(12), the RESET (12;6), the Joint bias test statistic (Engle and Ng, 1993) and the BDS test are all insignificant. The specification tests therefore suggest an appropriate bivariate model specification for spot prices and wind forecasts. The SNP projection gives access to the one-step-ahead bivariate spot and wind change densities $f_K\left(\tilde{y}_t \mid x_{t-1}, \hat{\theta}\right)$, conditional on $x_{t-1} = \left(\tilde{y}_{t-1}, \tilde{y}_{t-2}, \ldots, \tilde{y}_{t-L}\right)$. Figure 8 shows plots of the bivariate one-step-ahead density, conditional on x_{t-1}. We condition on a spot price movements from -60% to +60% (17 discrete impulse numbers) and wind forecast movements of -20% to +20% (15 discrete impulse numbers). The maximum and minimum are chosen based on observed market impulses for both spot price and wind forecast movements. The intervals between minimum and maximum values are chosen arbitrarily relative to on standard deviation of the unconditional densities. The plotted density matrix (dimensions are 17 (spot price impulses) x 15 (wind forecast impulses)) in Figure 8 displays interesting market dynamics[16]. The one-step-ahead densities at *t*+1 show highest densities (mean concentration) when spot prices and wind forecast impulses today (*t*) are small. That is, small movements to day suggest small movements the next day; large movements to day suggest large movements (wider density) the next day. Moreover, the diagonal densities from high price and low wind forecast movements (lower left corner) to low prices and high wind forecast movements (high right corner), both suggesting high contemporaneous negative correlation, clearly suggest higher densities relative to the two other corners of the matrix (positive correlation). The result suggests that bivariate impulses that produce negative correlation between price and wind forecast movements, establish higher and narrower days-ahead response densities (lower volatility). That is, positive (negative) prices and negative (positive) wind forecasts impulses at *t*. (contemporaneous negative correlation) clearly suggest a higher and narrower density matrix for the step-ahead spot prices. Figure 8 shows the importance of negative correlation between spot prices and wind for dynamic auction market strategies. In fact, the bivariate impulses that produce negative correlation between spot price and wind forecast movements suggest higher as well as narrower density responses. Importantly, the impulse-response functionals described in section 2, will give access to conditional mean and variance/covariance dynamics potentially valuable for market strategies.

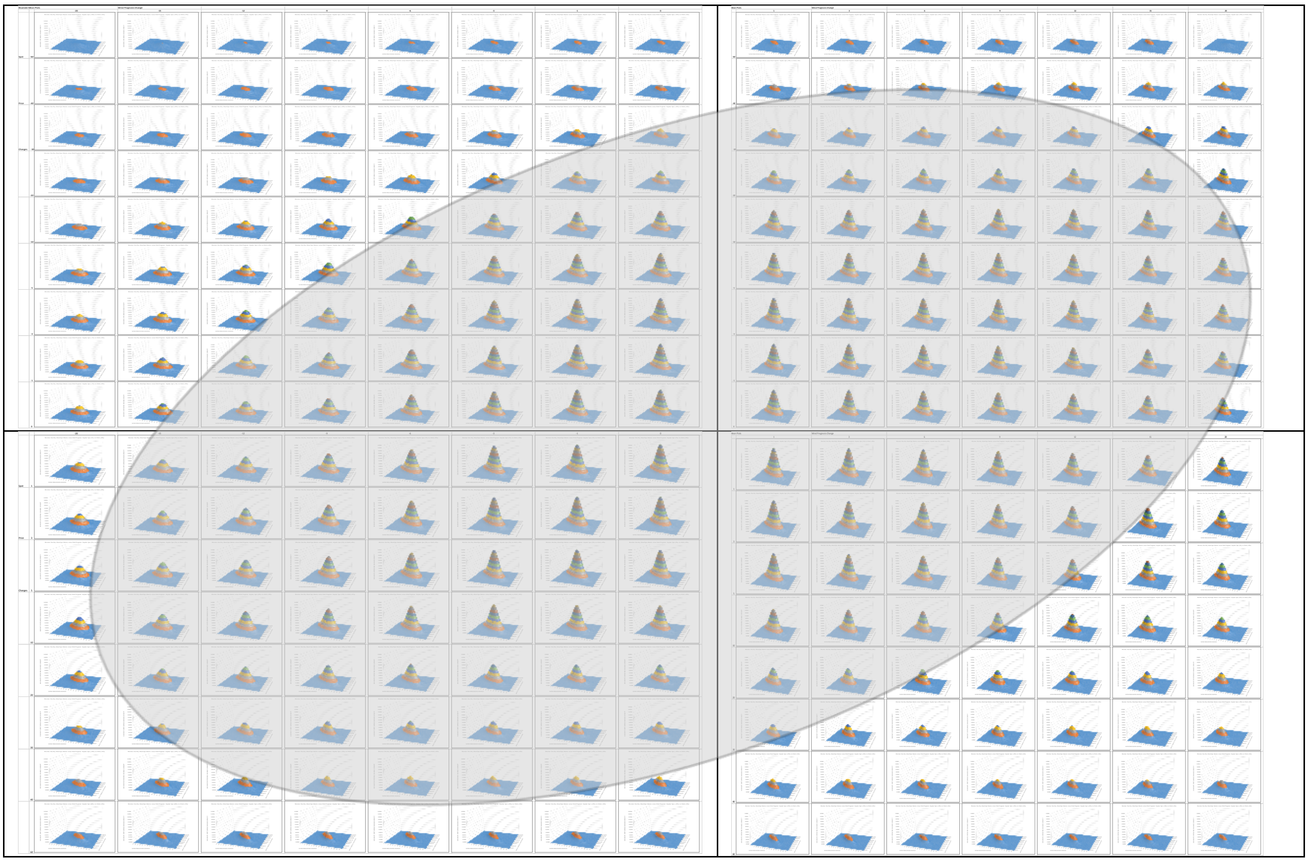

Figure 8. Conditional Bivariate Spot Price and Wind Forecast Mean Densities

[16] Note that the vertical axis (density) is constant in all 63 plots (7 x 9), while the horizontal axis (wind forecast) and depth axis (spot prices) are variable based on data.

5 The Impulse-Response Functions Analysis

The impulse-response analysis for the bivariate spot price and wind forecast movements give access to the conditional bivariate means and the variance/co-variance matrix *j*-days ahead. The computational implementation is quite trivial. From the daily BIC-optimal model, a wide set of bivariate impulses is established based on the bivariate unconditional mean and variances. From these bivariate price and wind densities, the step-ahead mean, variance and covariance are computed as described in section 2 above. The information is easily updated daily giving access and use of all historic information, enabling multi-step-ahead calculations of means, variance and co-variances. The model results can be used for dynamic forecasting of means and variance/covariances steps-ahead.

The SNP model's bivariate density $f_K\left(\tilde{y}_t \mid x_{t-1}, \hat{\theta}\right)$, which is described in section 4 above, is used for the impulse-response analysis. The methodology is similar to the step-ahead plots conditioned on x_{t-1} (Figure 8). Here, as described in section 2, we explicitly analyse j = 5 step-ahead mean and variance/covariance responses from bivariate spot price and wind forecast impulses. Note that Figure 8 above has suggested the importance of the bivariate spot price and wind forecast co-variances. These plots suggest that the bivariate mean and volatility densities are sensitive to the contemporaneous co-variance measures. Using the BIC-optimal model, the impulse-response analysis evaluates the daily bivariate mean and volatility responses from spot price and wind forecast impulses. The responses give therefore direct measures for the step-ahead means and variances/co-variances (forecasts). The impulse-response analysis below is threefold. Wind forecast figures should not show any effect from spot prices. Spot price mean and variance responses from spot-price and wind forecast movements are carefully monitored. Finally, we ca predict mean and volatility using spot price and wind forecast co-variances.

Firstly, we analyse the expectation that spot price movements should not influence wind forecast movements. Figure 9 and 10 therefore report wind forecast mean and volatility responses, respectively, from spot price and wind forecast impulses. For the mean responses, Figure 9 shows the fundamental model result that multiple-step-ahead wind forecast profiles is not significantly affected by spot price movements. By looking at the 95% confidence interval for -/+9% spot price impulses, the confidence intervals heavily overlap. Knowing that the 95% confidence intervals increases for higher spot price impulses, the wind forecast differences are insignificant. Note however, that the mean response is strongest (smallest) for contemporaneous positively (negatively) correlated spot price and wind forecast impulses. The analysis shows that spot price impulses from -60% to 60%, the responses are extremely small, while the wind forecast's own dynamics (-/+9 and -/+20%), influence the step-ahead wind forecast responses significantly harder (+/-10 and +/-25%). The wind forecast patterns show that high wind forecast impulses at time t (i.e. -20 and 20%) suggest an even stronger *reverse* wind response at t+1 (25 and -25%). Moreover, this wind impulse reversion at t+1 is marginally stronger for positive than for negative impulses at time t. For the wind forecasts volatility responses, Figure 10 reports wind forecast movement volatility responses for wind forecast impulses of -/+9% and -/+20%, over all spot price impulses. 95% confidence intervals are included for spot prices movements of -/+20% and wind forecast movements of -/+9%. These plots clearly indicate that wind forecast volatility is influenced from the size of spot price movements. These volatility effects emerge clearly from high/low spot-price impulse (look-ahead wind bias). Daily wind forecasts report multi-step-ahead (from step 2 and onwards the forecasts are very uncertain). Highly fluctuating wind forecasts may therefore suggest large spot price movements. Hence, larger spot price movement will therefore be influenced from multi-step-ahead wind forecast uncertainty, where the net effect is that contemporaneous large spot price movements reflect large wind forecast volatility. Note that a negative change in wind forecast impulses (-9% and -20%) has much higher effects on wind forecast variance than a positive wind forecast impulse (9% and 20%). Hence, a low wind forecast movement has higher step-ahead variance. The wind forecast volatility responses are not large and it should be expected that the volatility response differences are larger for when wind is low. The 3-dimensional (3D) plot in Figure 11 confirms these findings; linearly for the wind forecast mean and non-linearly for the wind forecast volatility. Spot price movements do not influence mean wind forecast responses. The Wind forecast response is unchanged over spot price impulses. For a large range of spot price movements (-30 - 60%), the wind forecast volatility seems not effected by the spot price movements. However, for extreme negative spot price movements (-60% and -40%), wind volatility seem to increase (meteorological multi-step-ahead forecasts from low wind). The main findings therefore show that spot price impulses do not affect wind forecasts and that we for the rest of the analysis, focus on spot price mean and volatility responses from the bivariate spot price and wind forecast impulses.

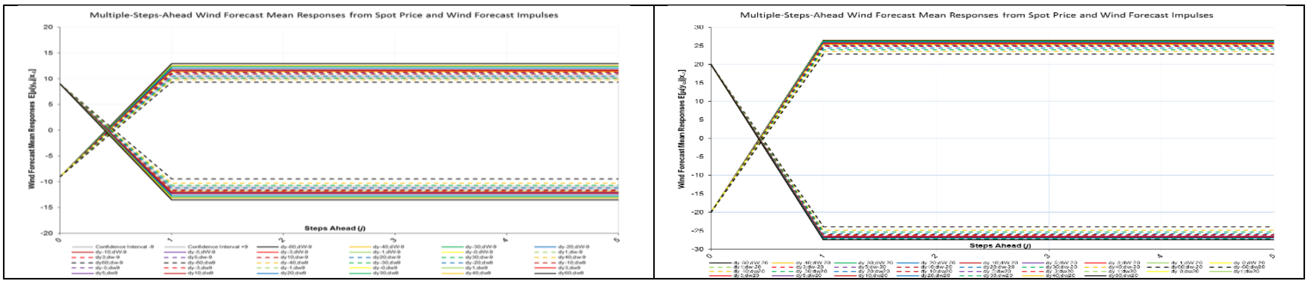

Figure 9. Wind Forecast Mean Responses for t +5 from varying Spot Price and Wind Forecast Impulses at t.

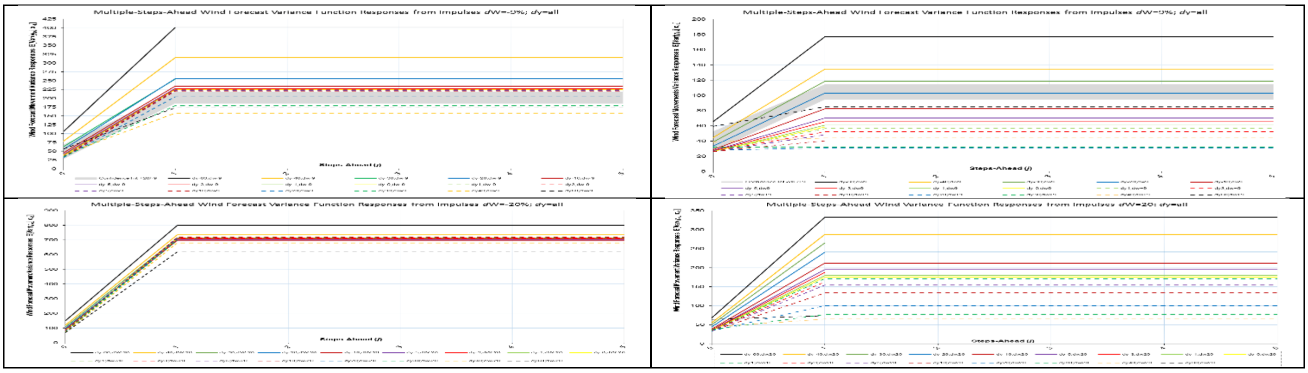

Figure 10. Wind Forecast Variance Responses for t +5 from Spot Price and Wind Forecast Impulses at t.

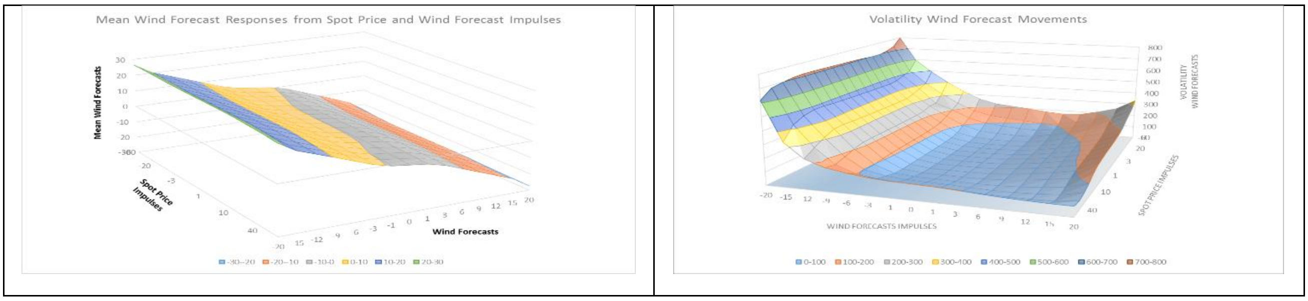

Figure 11. Wind Forecast Mean and Variance Responses for *t* +1 from Spot Price and Wind Forecast Impulses at *t*.

For the spot price responses, we start with plots of varying spot price impulses (wind forecast impulses) holding the wind forecast impulses (spot price impulses) constant. Figure 12 shows spot price responses to spot prices and wind forecasts impulses holding the spot price and the wind forecast, respectively, constant equal to zero. The spot price responses report overreaction and correction suggesting a negative correlation for the multi-step-ahead analysis. That is, large negative (positive) impulses at step 0 suggest positive (negative) responses one-step-ahead. The correction at one-step-ahead is close to symmetric. Figure 13 (14) reports the conditional mean for spot price changes (-60,…,60%) over two sets of wind forecast movements (-9% and 9%) and (-20% and 20%). Figure 13 shows that the negative (positive) wind movement of -9% (9%), shifts the spot price movements negatively (positively) from approximately 0 to -7.7 (0 to 8.5). With wind forecast of -9% (9%), only the extreme spot price of -60% (60%) report positive (negative) spot price response. For all other spot price impulses -9% (+9%), the spot price responses are positive (negative). Figure 14 shows the same result for constant wind forecast impulses of -20 and 20%. The negative (positive) wind movement of -20% (20%), shifts the spot price movements negatively (positively) from approximately 0 to -16.9 (0 to 18.2). With wind forecast of -20% (20%), all spot price impulses report negative (positive) spot price responses. That is, extreme values of wind forecast movements report spot price responses that do not change sign over the whole range of spot price impulses. Figure 13 also reports spot price response confidence intervals for all spot price and -9% and 9% wind forecast impulses. The 95% confidence intervals for the responses are quite wide but they are clearly different from zero. Moreover, note that the spot price reversion is strongest when spot prices and wind forecasts are negatively correlated. The surface plot in Figure 15 summarizes our findings for spot price mean responses from spot price and wind forecast impulses. The plot reports six surface areas for the spot price mean responses. For values of spot price and wind forecast impulses that show negative contemporaneous co-variance, our results suggest absolute spot price responses with potential profit for dynamic market positions (long/short positions). Moreover, the negative spot price response serial correlation (for step 0 and 1) suggest that large negative (positive) spot price responses from positive (negative) spot price impulses, suggest negative (positive) wind forecast responses. That is, when the contemporaneous co-variance between spot prices and wind forecast impulses are highly negative at time *t* (i.e. dy_t=60%; dW_t=-20% and dy_t=-60%;dW_t =20%) the mean spot price response at *t*+1 are clearly different from zero. Because of negative serial correlation for one-step-ahead spot price movements, the response has opposite sign of the impulse. In contrast, when the correlation between spot prices and wind forecasts is contemporaneously positive at time *t* (i.e. dy_t=-60%; dW_t=-20% and dy_t=60% ;dW_t =20%) the step-ahead price changes at *t*+1 seem to response revert to a value much closer to zero (and induce lower volatility).

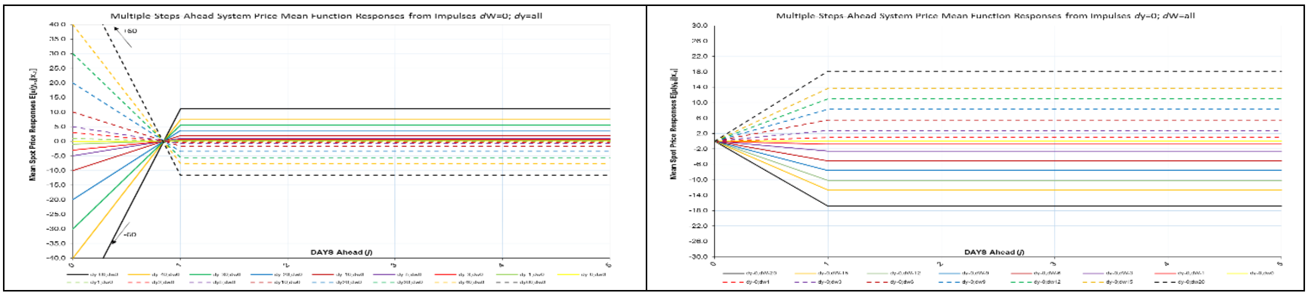

Figure 12. Spot Price Mean Responses for *t* +5 from Spot Price Wind Forecast Impulse at *t*.

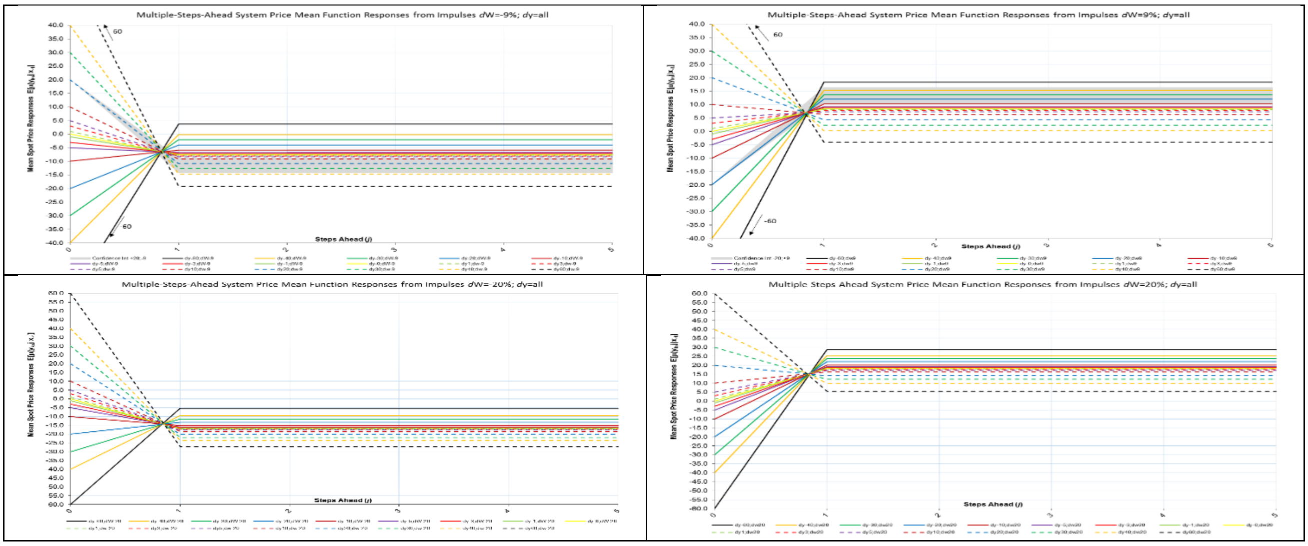

Figure 13. Spot Price Mean Responses for *t*+5 from all Spot Price Impulses and Wind Forecast Impulses at *t*.

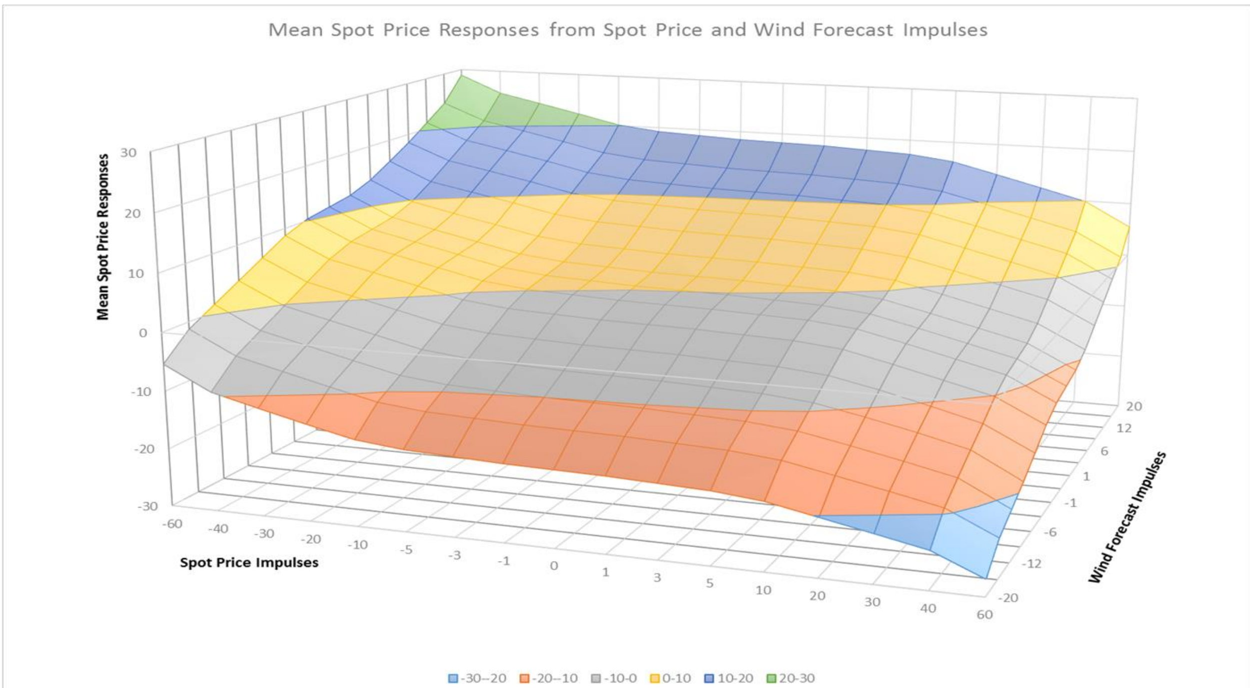

Figure 14. Spot Price Mean Responses for *t* +1 from all Spot Price and Wind Forecast Impulses at *t*.

The spot price and wind forecast co-variance and the co-variance effects on the one-step-ahead spot prices seem therefore related for the negative co-variance. The normal market situation suggests more (less) wind means more (less) supply of power, which again suggest lower (higher) spot prices. Now, referring to the wind forecast responses, the market expects one-step-ahead wind reversion/correction. Hence, high negative co-variance means spot price response reversion/correction. That is, in a high negative covariance regime, positive (negative) spot price impulses suggest negative (positive) spot price responses.

For the spot volatility responses, we start with plots of the varying spot price impulses (wind forecast impulses) holding the wind forecast impulses (spot price impulses) constant. Figure 15 shows spot volatility responses from spot price and wind forecast movements holding the other variable, wind forecast movements, constant equal to zero. From these two plots, we find that spot price volatility is influenced by both spot price and wind forecast impulses; more so for the spot price movements. For small values of both spot price and wind forecast movements, the step-ahead volatility decreases. The univariate spot price volatility seems to increase from -/+10% movements and is quite symmetric. For the wind forecast movements the spot price variance increase is much smaller. However, the spot price variance influence is much more asymmetric. The spot price variance increases more for positive wind forecast impulses than negative impulses. Figure 16 shows the spot price variance responses for all spot price impulses and for wind forecast impulses of -9% and 9% (-20% and 20%). Figure 16 shows that that a positive wind forecast impulse influences spot prices marginally more than negative wind forecast impulses. However, the 95% confidence interval is wider. However, note from these plots the importance of negative co-variance. Figure 18 show a 3-dimensional (3D) plot for the spot price variance response from the bivariate spot price and wind forecast impulses. The plot shows that variance is calm for a wide range of spot price and wind forecast impulses. For both large negative and positive spot price impulses, the spot price variance response increases quickly. Moreover, note the increase for wind forecasts impulses that have a negative covariance with spot price impulses.

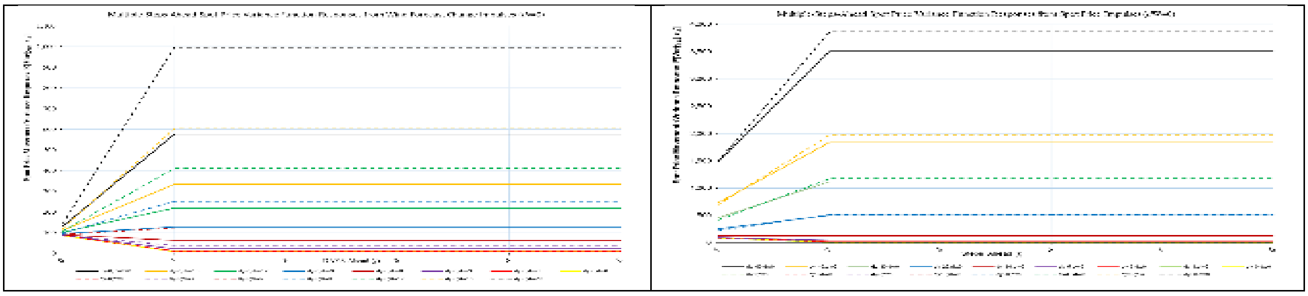

Figure 15. Spot Price Variance Responses for *t* +5 from Spot Price and Wind Forecast Impulses at *t*.

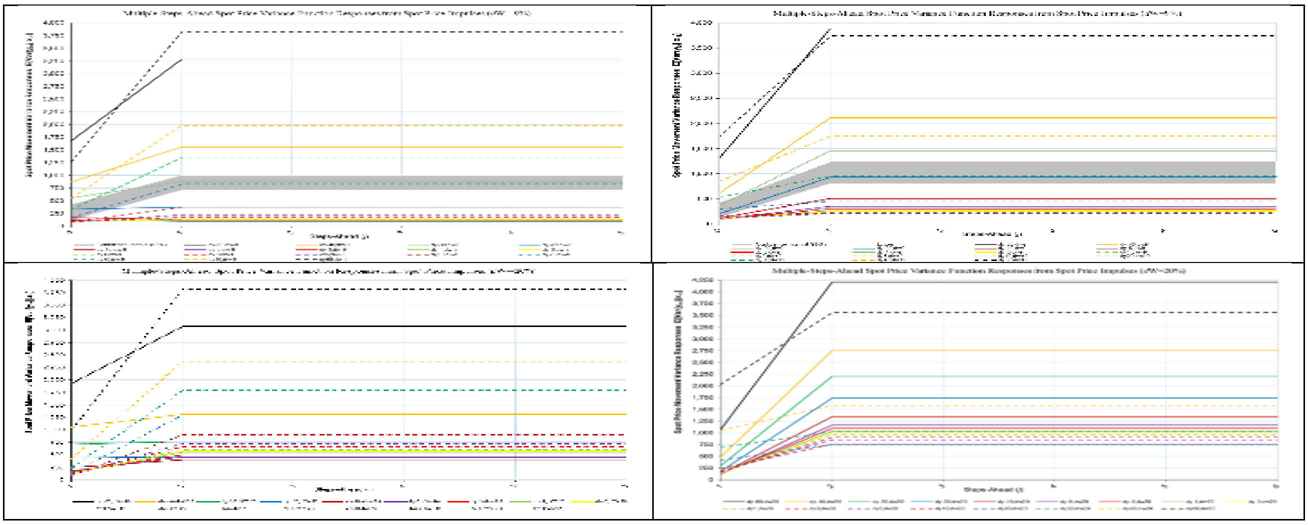

Figure 16. Spot Price Variance Responses for *t*+5 from Spot Price and Wind Forecast Impulses at *t*.

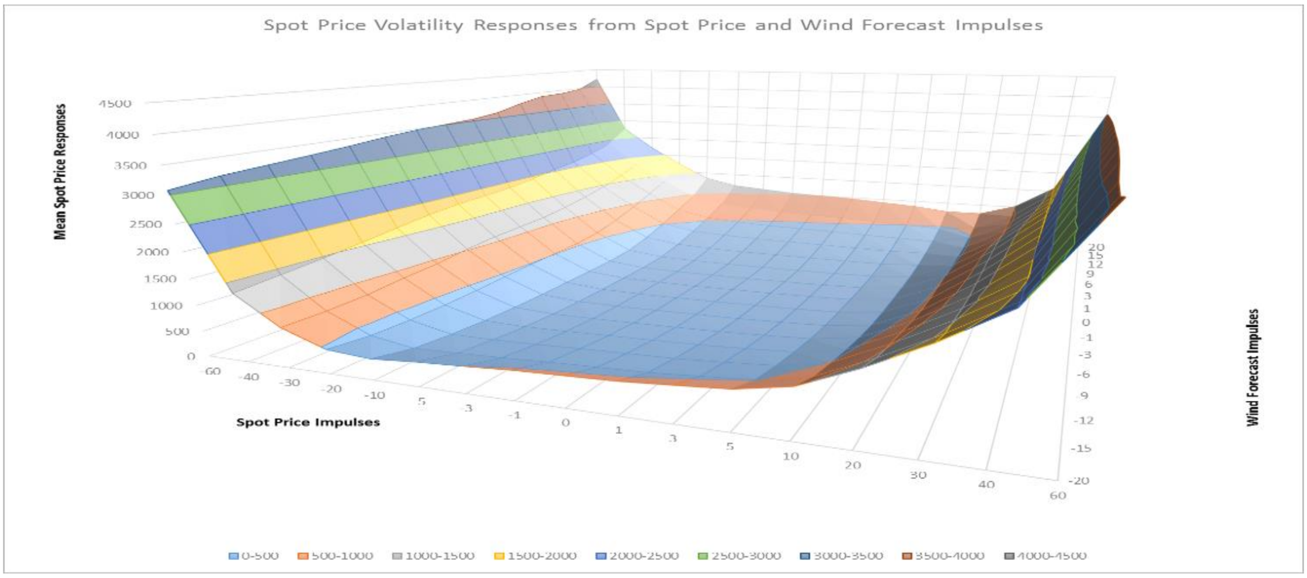

Figure 17. Spot Price Variance Response for *t* +1 from Spot price and Wind Forecasts Impulses at *t*.

From the above spot price analyses, the spot price and wind forecast co-variances are important for our findings. Figure 18 shows that the spot price and wind forecast co-variance becomes more negative from spot price impulses relative to wind forecast impulses. Moreover, the spot price impulses report quite symmetric co-variance responses relative to wind forecast impulses. Figure 19 reports spot price and wind forecast co-variance responses for spot price impulses and wind forecast impulses -9% and 9% (-20% and 20%). First, there is no sign of positive co-variance. High negative co-variance is reported for both high and negative spot price impulses, but more so for spot price and wind forecast with opposite signs. This is quite clear when we look at Figure 20 and the extreme impulse values of both spot price and wind forecast is reported. Note that there is no asymmetry for the co-variances. Figure 21 show a 3-dimensional (3D) plot of Spot price and wind forecast co-variance from spot price and wind forecast impulses. The co-variance becomes large negative when the spot price and wind forecast impulses moves in opposite directions. For small impulse movements for both price and wind, the covariance is quite stable and small.

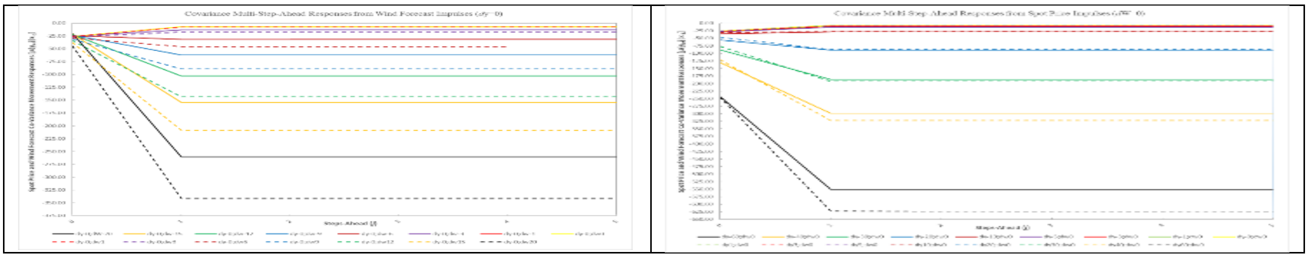

Figure 18. Spot Price and Wind Forecast Co-Variance Responses t+5 from Spot Price and Wind Forecasts Impulses at t.

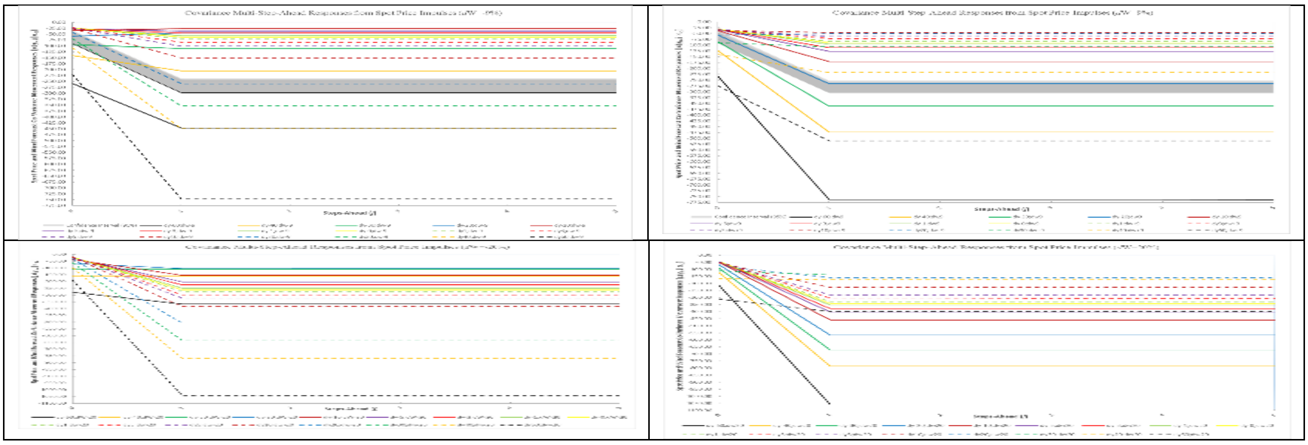

Figure 19. Spot Price and Wind Forecast Co-Variance Responses t+5 from Spot Price and Wind Forecasts Impulses at t.

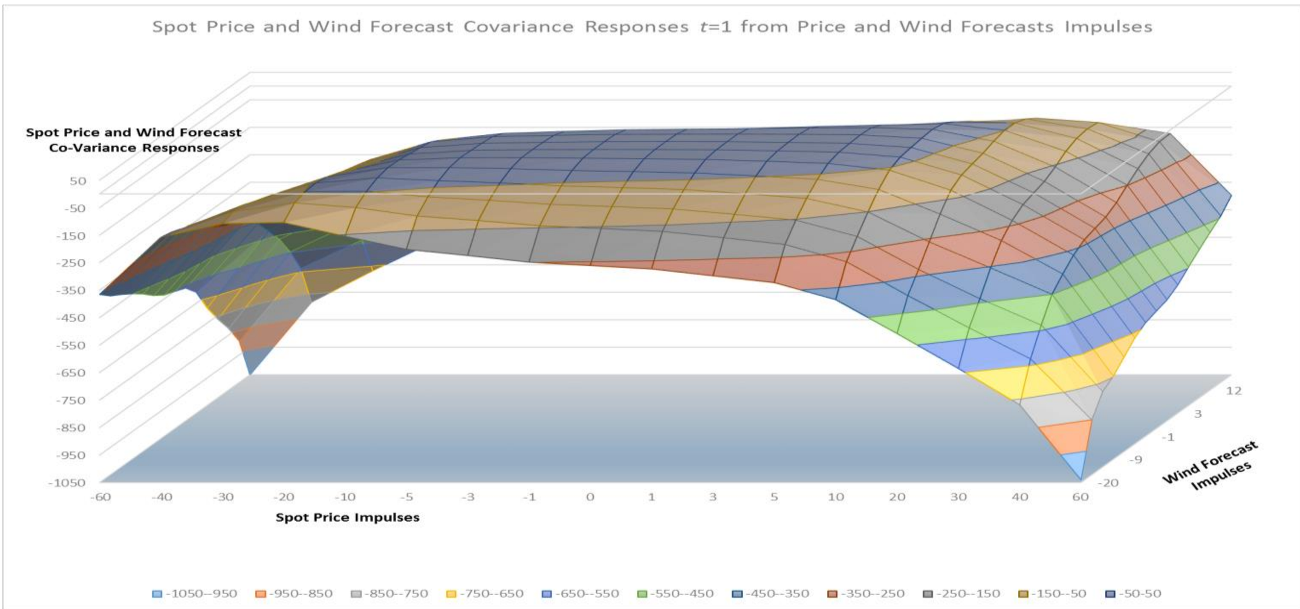

Figure 20. Spot Price and Wind Forecast Co-Variance Responses t +1 from Spot Price and Wind Forecasts Impulses at t.

Finally, we use the model's co-variance results to predict spot price means and volatility one-step-ahead. Figure 21 show the prediction for one-step-ahead spot price means and Figure 22 shows the one-step-ahead prediction for spot price volatility. The solution is dynamic with a continuous need to update information on daily basis. From calculated covariances the positions for the mean and volatility at t+1 is readily available for market participants. A dynamic trading position involving the available one-day forward/future market can be used to establish daily dynamic market positions. However, there is a need for implementation of the BIC-optimal conditional model and the post-estimation impulse-response analysis is needed on daily basis to establish daily dynamic market positions.

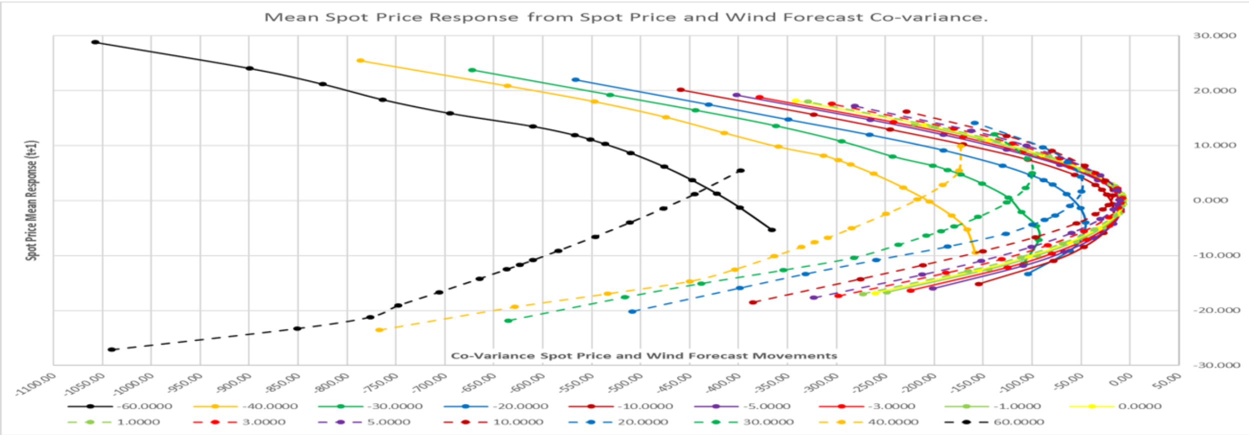

Figure 21. Forecasting Spot Price Changes for *t* +1 across Co-variance Spot Price and Wind Forecast Movements at *t*.

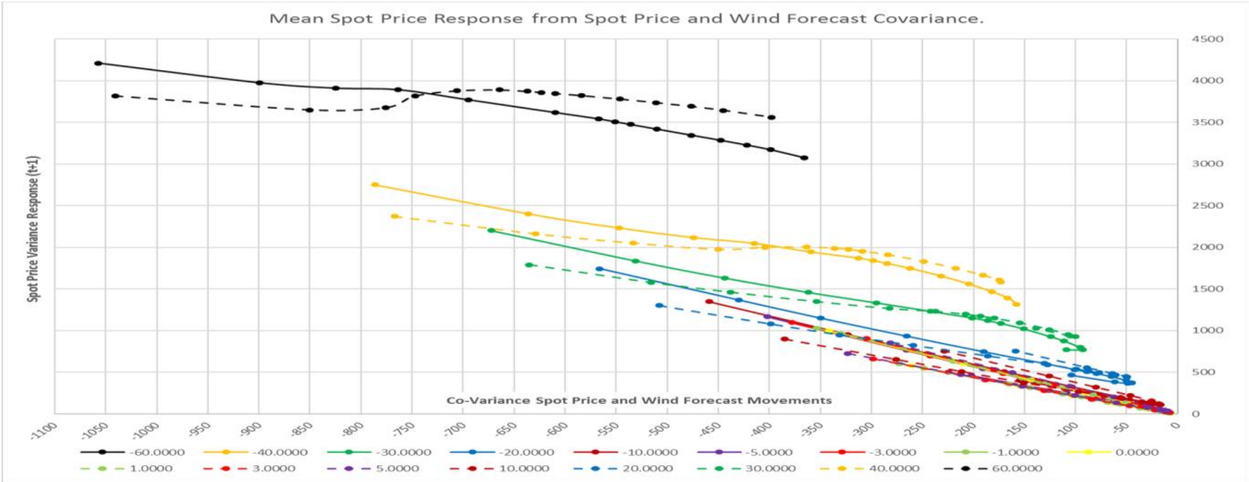

Figure 22. Forecasting Spot Price Volatility for *t* +1 across Co-variances Spot Price and Wind Forecast movements at *t*.

6 Summary and Conclusions

We have modelled and estimated an bivariate ARMA-GARCH-in-Mean model specification for the conditional mean and variance for the so-called system price and wind forecasts in the Nordic electric power market for the period January 2013 to January 2017. The two series are adjusted for systematic seasonal, trend and scale effects for stationarity. The paper applies a bivariate semi-non-parametric procedure employing expansions in Hermite functions to approximate the conditional density of the bivariate spot price and wind forecast movements. The paper reports direct computation of functionals of the fitted bivariate density. Moreover, computational accessible sample paths, which can be used to compute nonlinear functionals of the density, can be obtained by Monte Carlo integration. The methodology also uses simulated sample paths to set bootstrapped sum-norm confidence bands on the computed nonlinear functionals.

The impulse-response analysis give access to a vector of means and variance/covariance matrices. Our findings suggest that the spot price and wind forecast co-variances motivate expectation regularities. From a *BIC* optimal model and a set of impulses, the impulse-response analysis reports regularities that suggest potential dynamic market strategic behaviour. Since wind together with other renewables, will be an increasing input factor for electricity production in the time to come, the dynamic spot price and wind forecast co-variance estimates will be important information for daily market positions in the Nordic/Baltic Electricity Market.

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